

# 緩和系高ベータプラズマの研究とQUEST計画への期待

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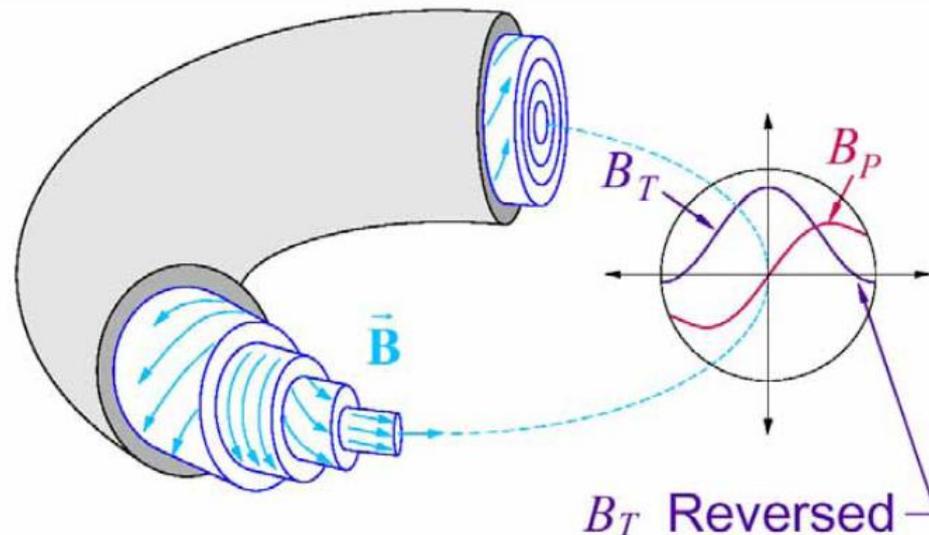
- RFP磁場配位について
- RELAX装置と運転領域
- Single Helical (SH) RFP配位とRELAXの特徴
- SH RFP緩和モデル
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第15回QUEST研究会  
2018年7月20日 九州大学応用力学研究所

RFP磁場配位について

# Reversed field pinch magnetic configuration

- Magnetic fields are produced primarily by plasma current.
- Weak external magnetic field is sufficient.
  - favorable to fusion reactor
  - strong magnetic shear, with (generally) weak toroidal effect
    - complementary to tokamaks and stellarator/heliotron
- RFP plays important roles in self-organization and nonlinear MHD studies



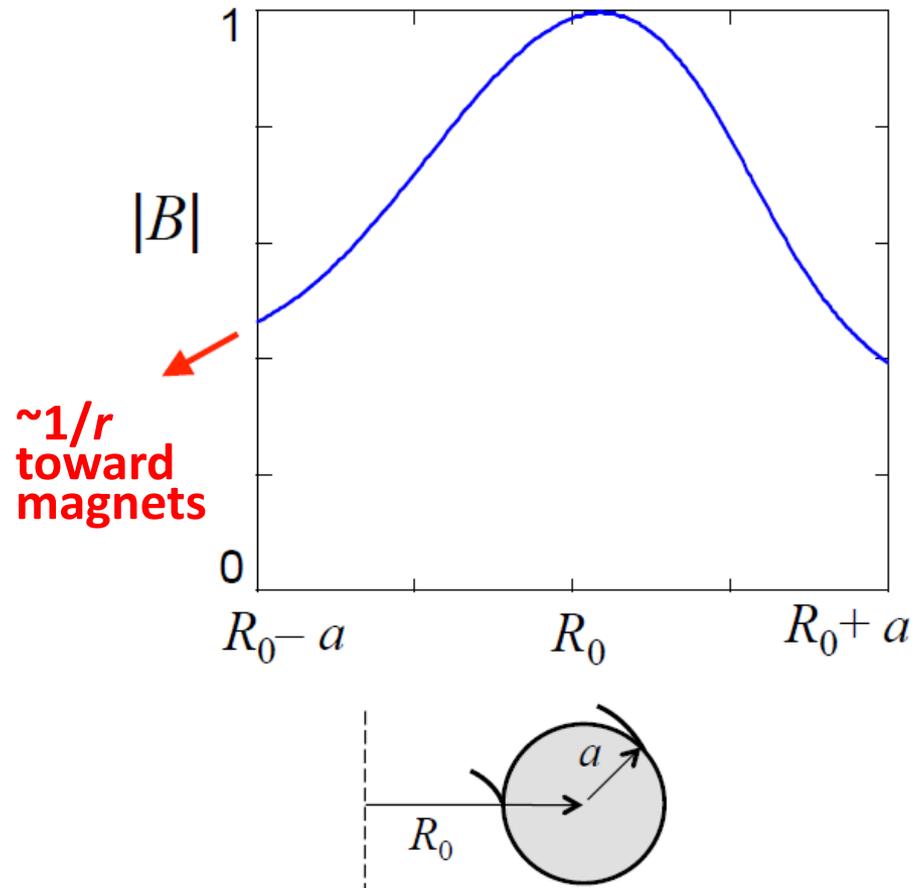
# Advantage of RFP as a fusion reactor: Magnetic fields are primarily inside the plasma, with weak external field for confinement

Normal conducting coils could be used because of the weak magnetic field at the coil locations

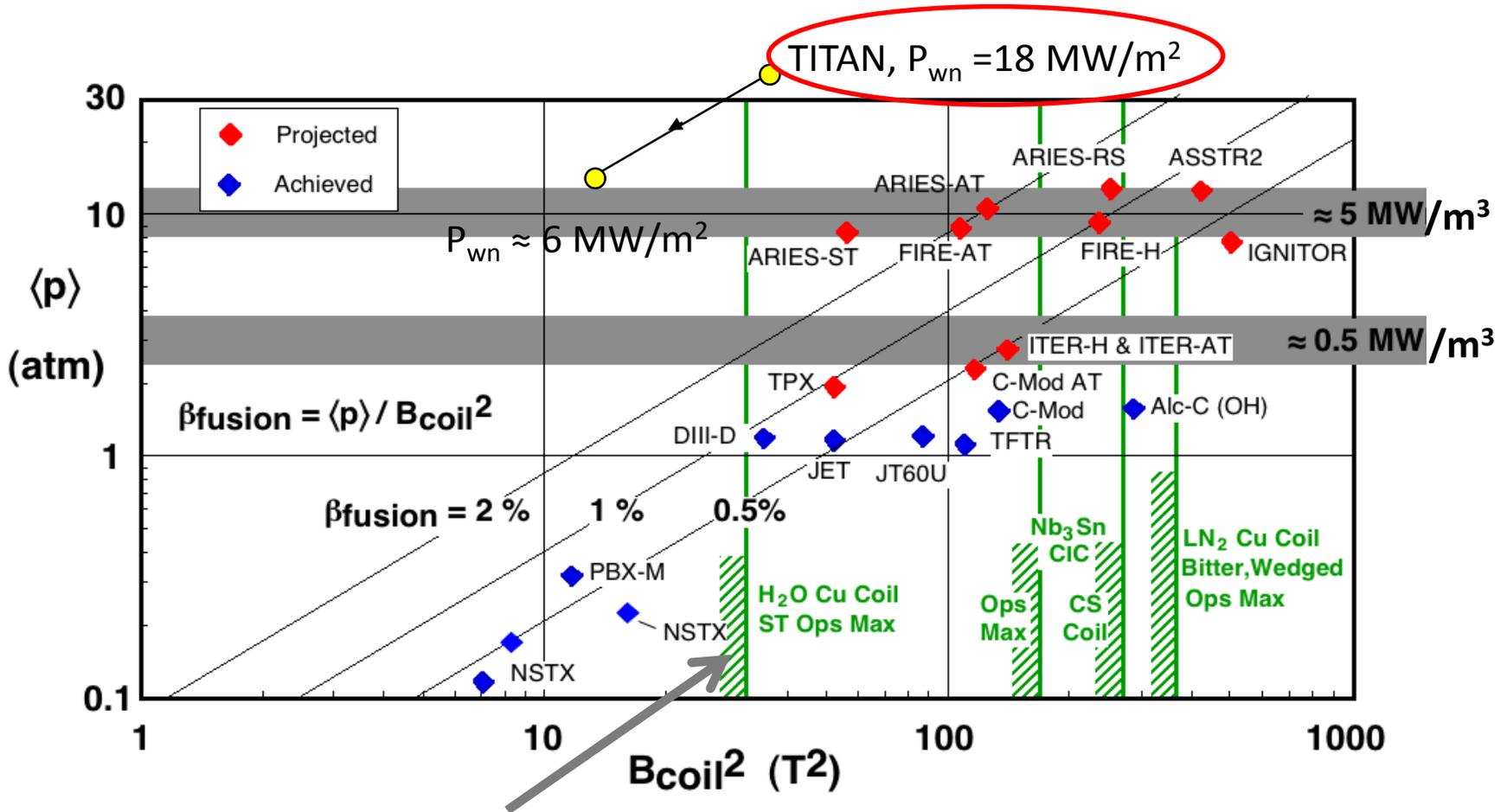
- magnetic pressure at the coils is 1/10 of that of tokamak
- high reliability of the system and easy maintenance

Very high current density

- Ohmic heating to the fusion reactor regimes is possible
- No need for plasma-facing auxiliary heating system
- High density limit ( $n_G \sim I_p/a^2$ )



# Advantage of weak external field

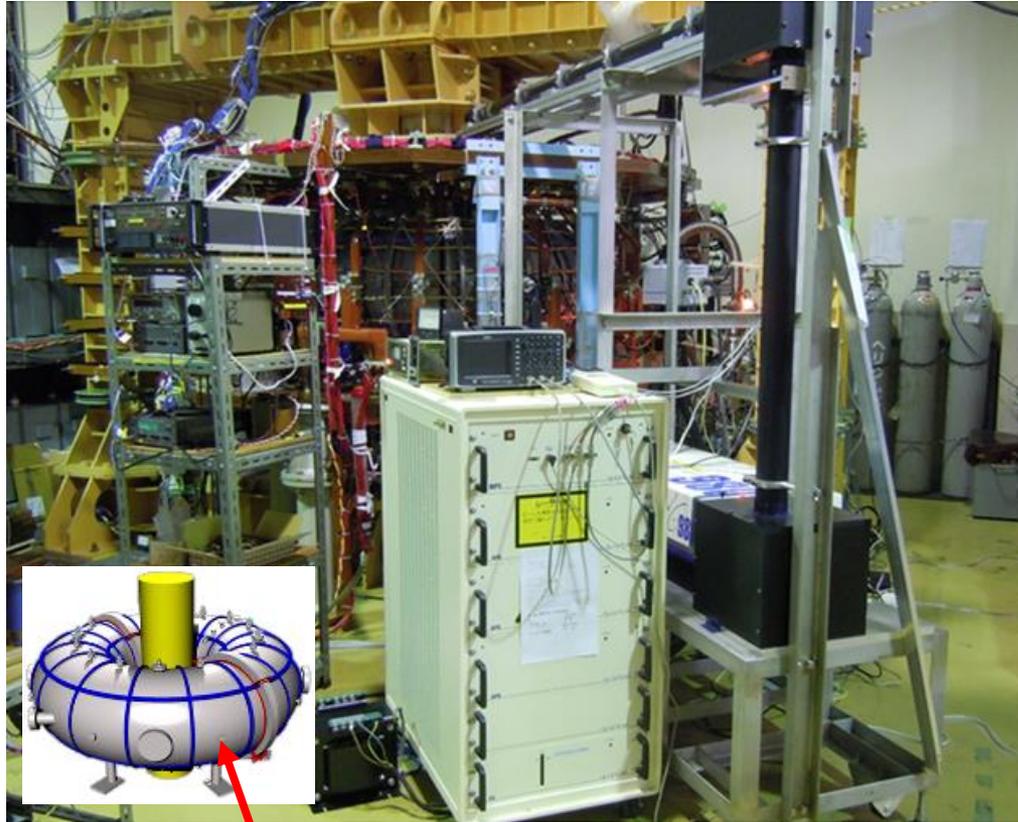


**Water-cooled Copper coils will work**

(Courtesy of John Sarff)

## RELAX装置の概要

# REversed field pinch of Low-Aspect-ratio eXperiment



$R/a = A = 2$   
(0.51 m/0.25 m)

Vacuum vessel without a  
conducting shell  
→ resistive wall boundary

Operation region and  
plasma parameters:

$$I_p < 125 \text{ kA}$$

$$n_e = 10^{18} \sim 10^{19} \text{ m}^{-3}$$

$$T_e(0) \sim 100\text{-}200 \text{ eV}$$

$$\beta_{pe0} \sim 5\text{-}15\%$$

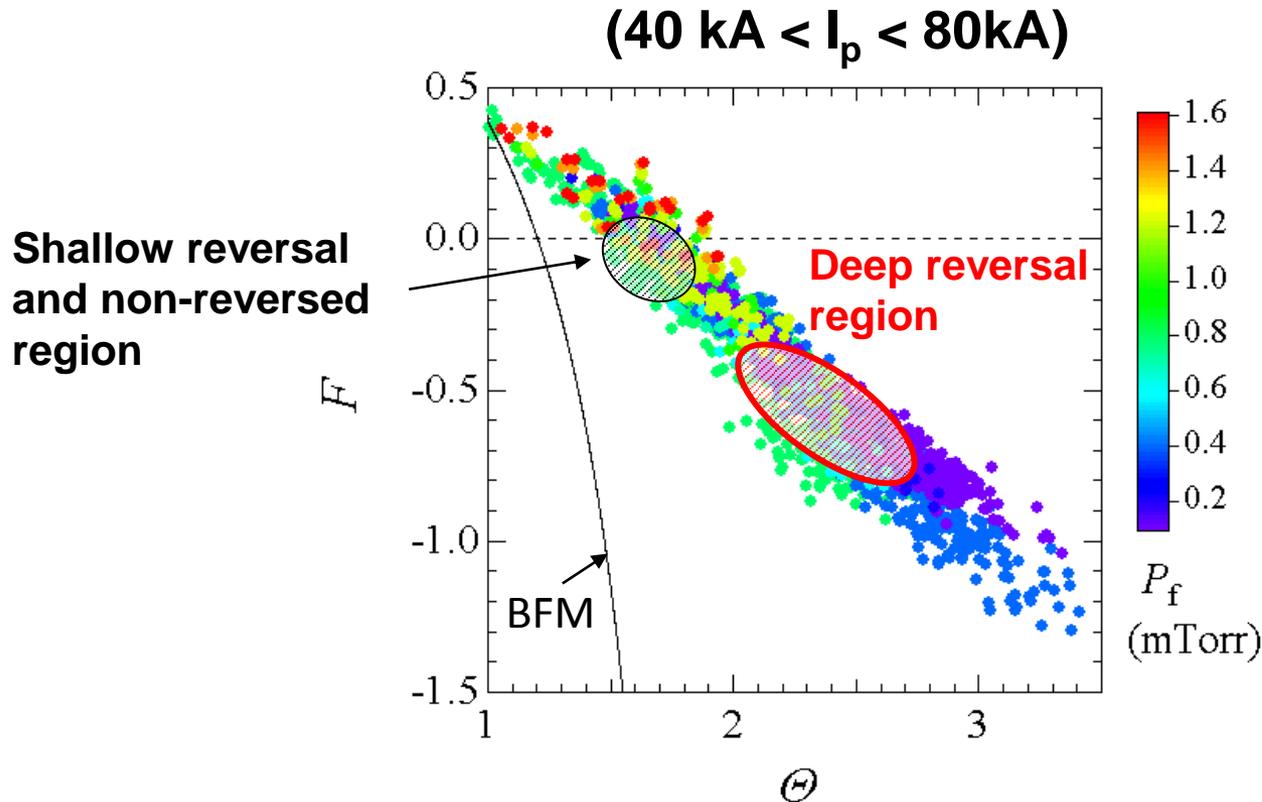
$$\tau_D > 3 \text{ ms}$$

64 saddle coils  
for active MHD  
control

Research objectives:

- geometrical optimization of RFP
- bootstrap current issues
- MHD with resistive wall boundary

# Stable plasmas are realized over a wide operation region in ( $\Theta$ , $F$ ) space



$$\Theta = \frac{B_p(a)}{\langle B_t \rangle}$$

$$F = \frac{B_t(a)}{\langle B_t \rangle}$$

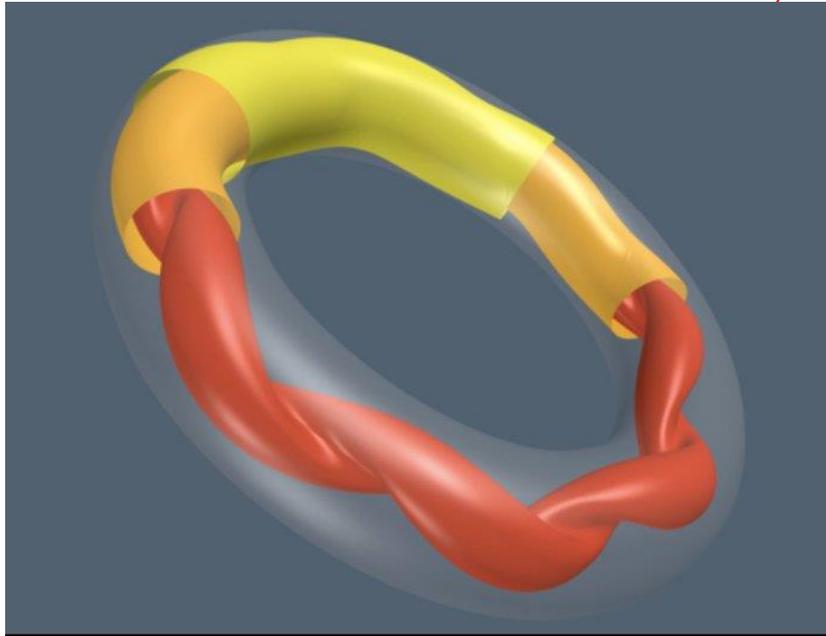
(Ikezoe et al., PPCF 2012)

- In shallow reversal region,
  - Quasi-periodic Quasi-Single Helicity (QSH) or Single Helical RFP state tends to be realized
- In deep reversal, high- $\Theta$  region,
  - Amplitudes of resonant modes are suppressed with broad spectrum
  - SXR emission increases, indicating improved plasma performance

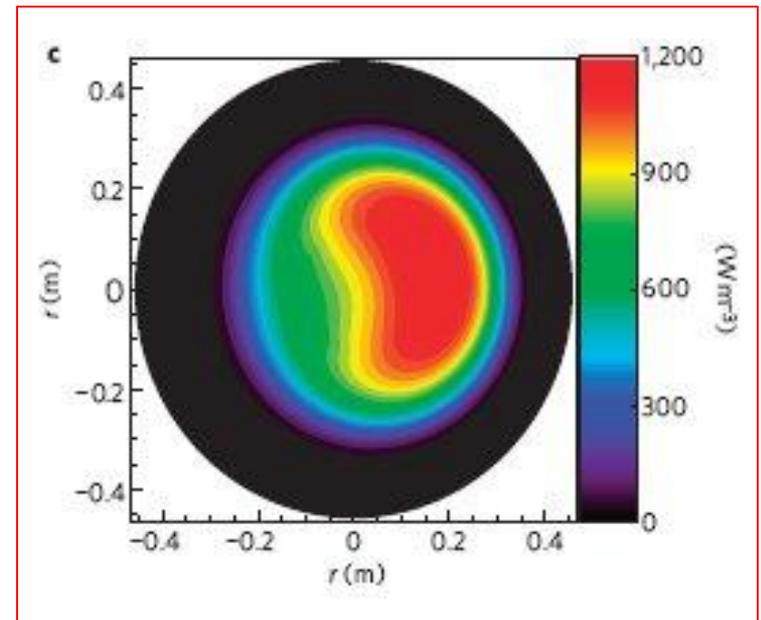
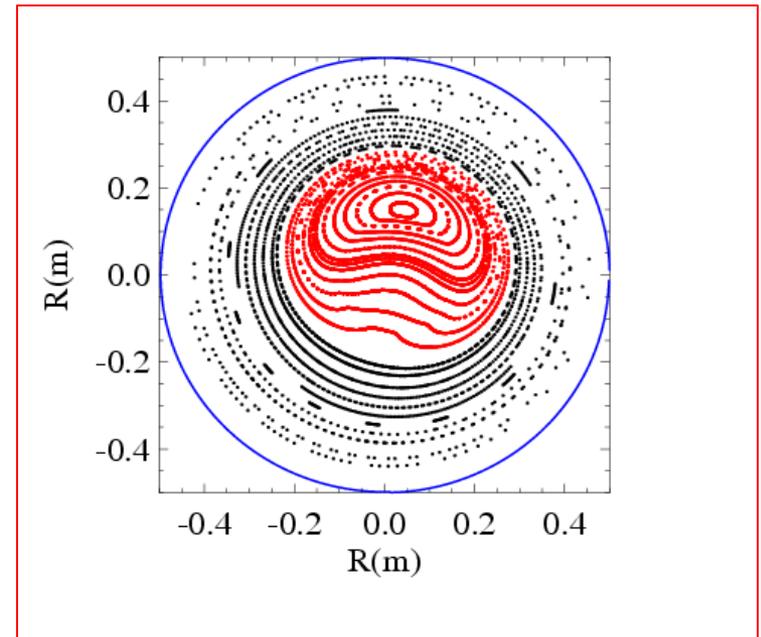
Single Helical (SH) RFP 配位

# Single Helical (SH) RFP state

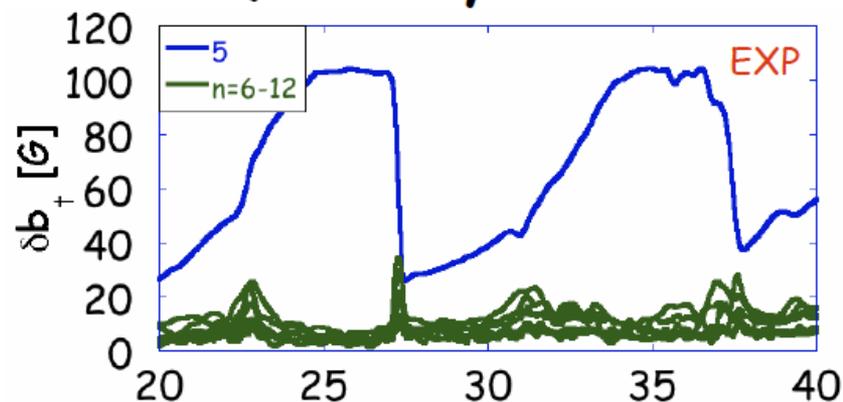
Lorenzini et al., Nature Phys. 5, 570 (2009)



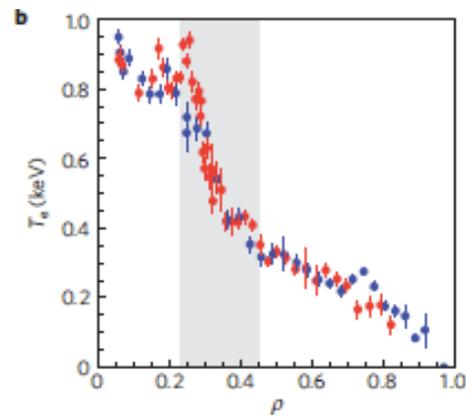
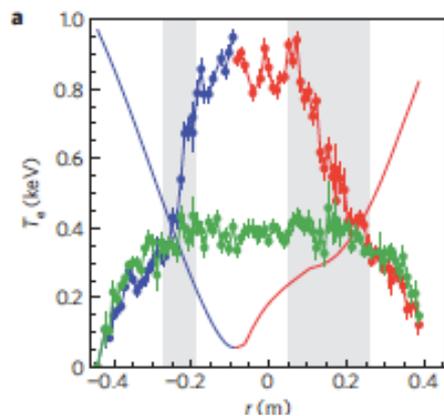
- Significant fraction of the plasma volume involved
- Theoretical explanation  
*Escande et al., PRL. 85, 3169 (2000)*



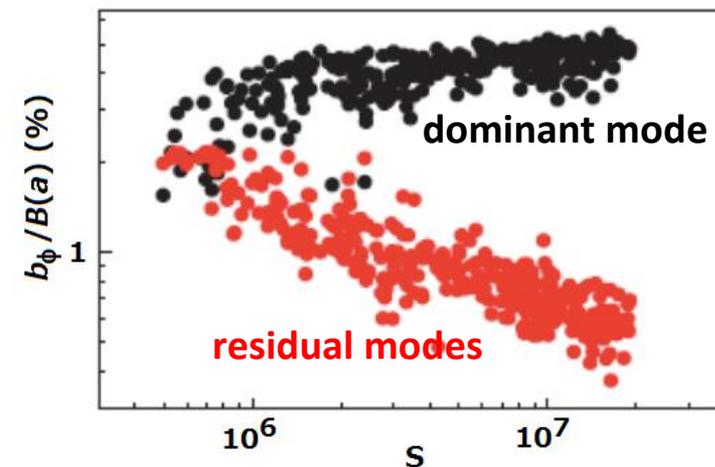
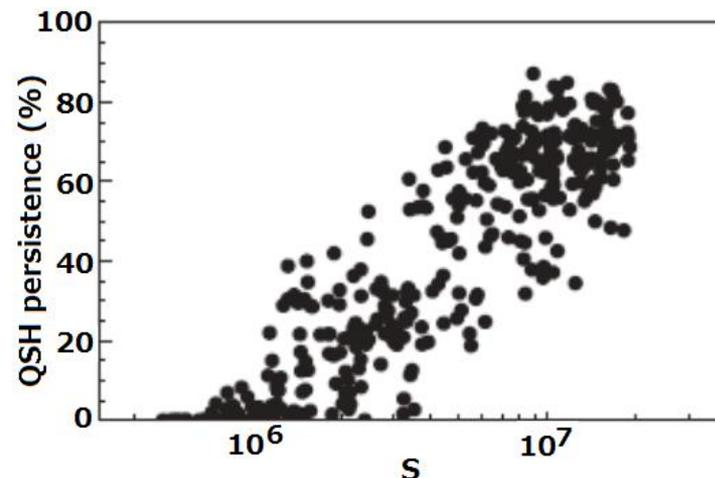
# Characteristics of SH states in RFX-mod and MST



Periodic or intermittent transition to QSH (MST)

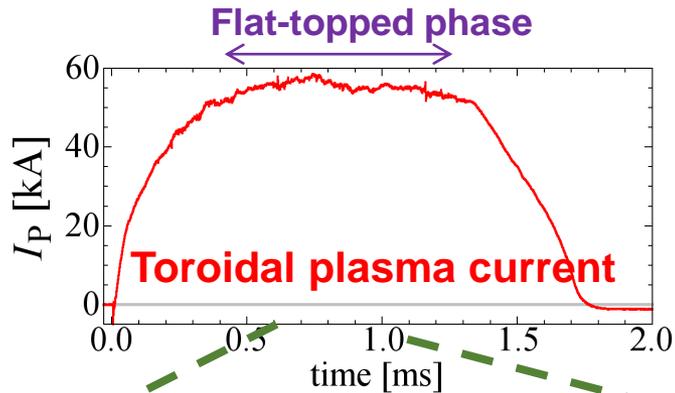


Formation of helical hot core with electron thermal transport barrier (RFX-mod)

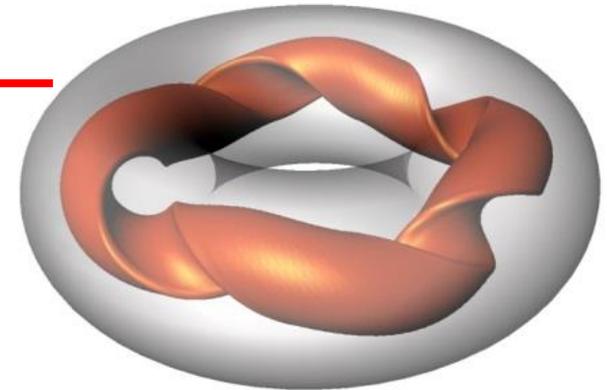
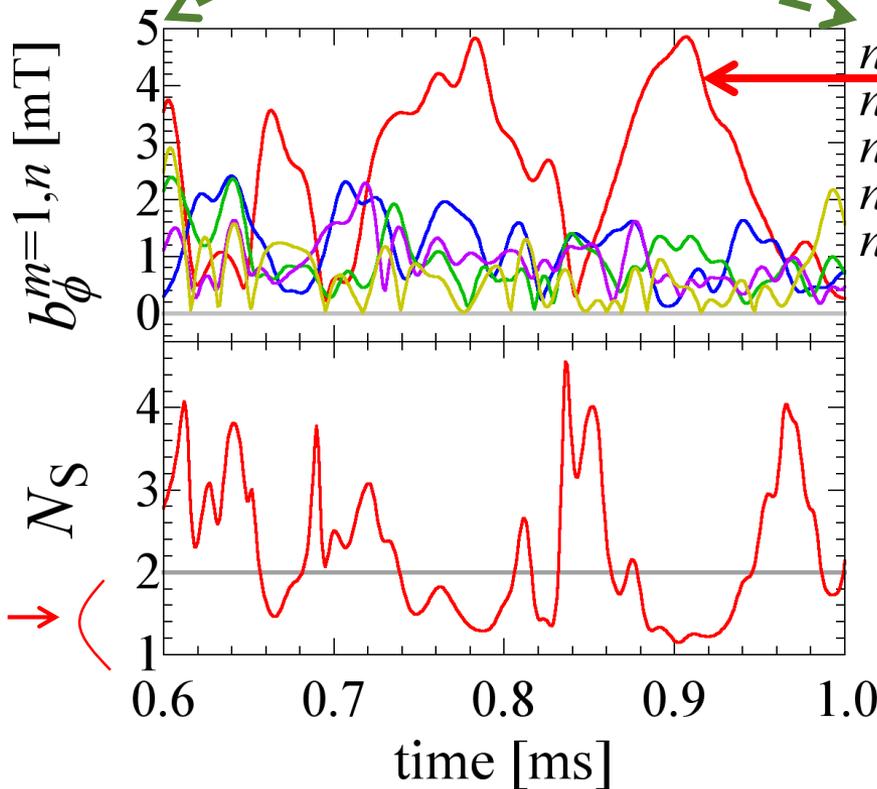


Purer QSH with higher persistence for higher  $S$  (RFX-mod)

# Example of QSH in RELAX from edge magnetic fluctuation



- QSH defined by  $N_s < 2$  can be confirmed from edge magnetic fluctuation spectrum.
- QSH appears quasi-periodically.

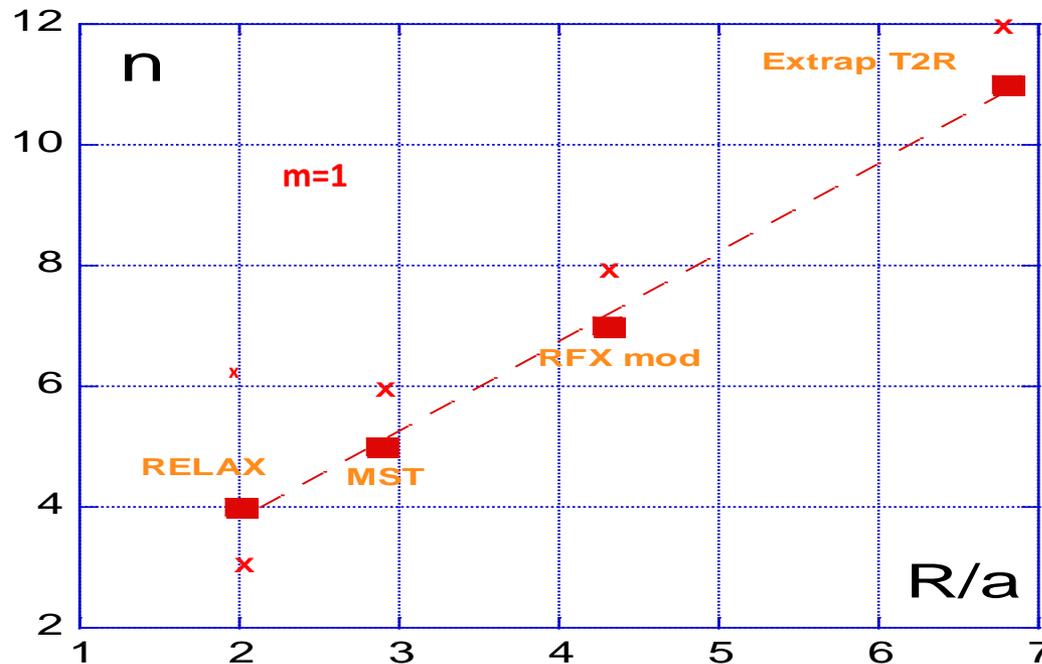


Spectral index  $N_s$ :  
a quantitative measure for QSH

$$N_s = \left[ \sum_{n=3}^8 \left( \frac{(b_{\phi}^{1,n})^2}{\sum_{n=3}^8 (b_{\phi}^{1,n})^2} \right)^2 \right]^{-1}$$

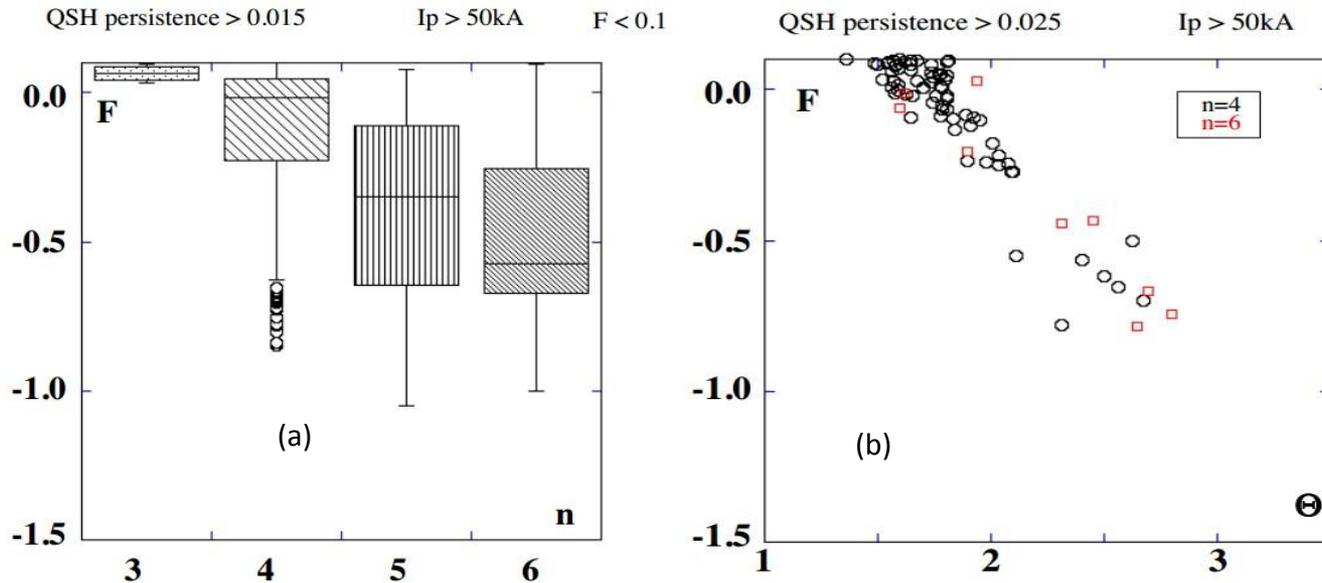
(Oki et al, PFR 2012; FST 2013)

## Dominant n numbers for the SH states at different aspect ratios



- Reversed field pinches operate in a wide range of aspect ratios ( $A=R/a$ ), from 2 to almost 8.
- At all these different  $A$ 's single helical states are observed with different toroidal periodicities ( $n$  numbers) and with poloidal mode number  $m=1$ .

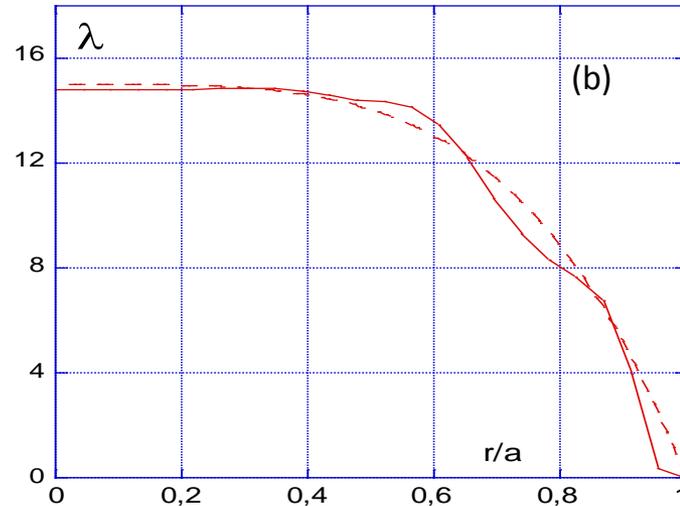
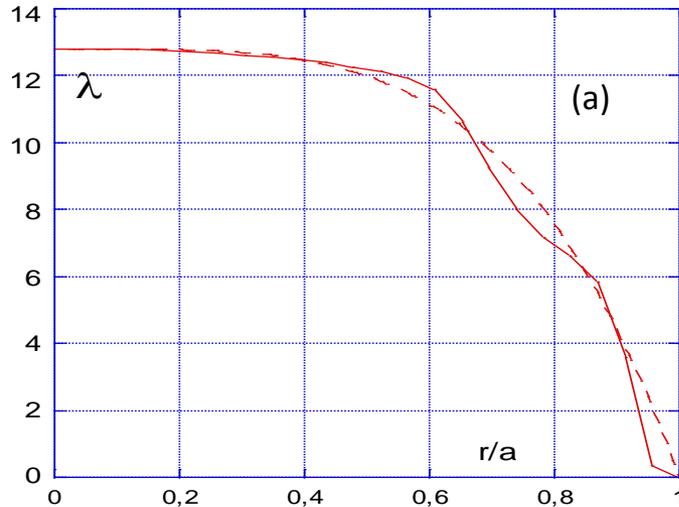
# Statistical characteristics of Single Helical RFP in RELAX



Dominant toroidal mode number  $n$  at different  $F$  values. Each box encloses 50% of the data with the median value (line).

- At shallow reversal the dominant modes are  $n=3,4$  while at deep reversal  $n=5,6$  could be dominant.
- When the probability of 3% is set as a threshold, only  $n=4$  and marginally  $n=6$  modes persist.
- The helical structure is not long-lived, and the most robust is the  $n=4$  mode.

# Parallel current density profiles in shallow- and deep-reversal regions



$\lambda$  profile (in  $\text{m}^{-1}$ ) vs. normalized radius for shallow F (a) and deep F (b) values in RELAX. Dashed line represents a fit with  $\lambda = \lambda(0)[1 - (r/a)^4]$ .

- Parallel current density is defined by  $\lambda = \mu_0 \frac{J \cdot B}{B^2}$
- Two  $\lambda$  profiles are deduced by the equilibrium reconstruction in RELAX at shallow and deep F values.
- The profile looks like almost the same for the two F values.

# Single Helical (SH) RFP 配位に関する理論モデル

# Single Helical, ideal wall, relaxation theories

- Mode Control has produced a quasi-ideal boundary and SH occurrence has increased (RFX-mod, Extrap T2R, RELAX)
- It is known that Taylor's relaxation requires an ideal wall for helicity and flux conservation
- Taylor's theory is **not able to** predict the correct helicity: because the on-axis modes are stable for perfectly flat  $\lambda$  profile
- A theory with **non-flat  $\lambda$  for a plasma dominated by a Single Mode** already exists

## Questions:

- i) Is this theory predicting correctly the RFP states ( $F$  and  $\Theta$ )?
- ii) Is this theory able to predict the observed scaling with  $R/a$  ? and with  $F$ ?

A. Bhattacharjee, R. L. Dewar, and D. A. Monticello, Phys. Rev. Lett. 45 347 (1980).

A. Bhattacharjee and R. L. Dewar, Phys. Fluids 25, 887 (1982)

A. Bhattacharjee, R. L. Dewar, A. H. Glasser, M. S. Chance, and J. C. Wiley, Phys. Fluids 26, 526 (1983)

## Relaxed states with a single mode

Beside Taylor's invariants,  $K_o$  and  $F$  (toroidal flux) a new invariant related to SH relaxation is introduced:

$$K_1 = \frac{1}{2} \int_V \chi^d \mathbf{A} \cdot \mathbf{B} dV$$

where :  $\chi = q_s \Psi - \Phi$

is the helical flux ,  $q_s = \frac{m}{n}$  , and  $\Psi$ ,  $\Phi$  are the poloidal and toroidal magnetic fluxes

→ The relaxed minimum energy states satisfy the simple relation:

$$\mathbf{J} = \frac{\lambda_0(2+d)}{2} \chi^d \mathbf{B}$$

- The wall is assumed to be IDEALLY CONDUCTING
- $d$  is a positive integer (related to the invariance request)

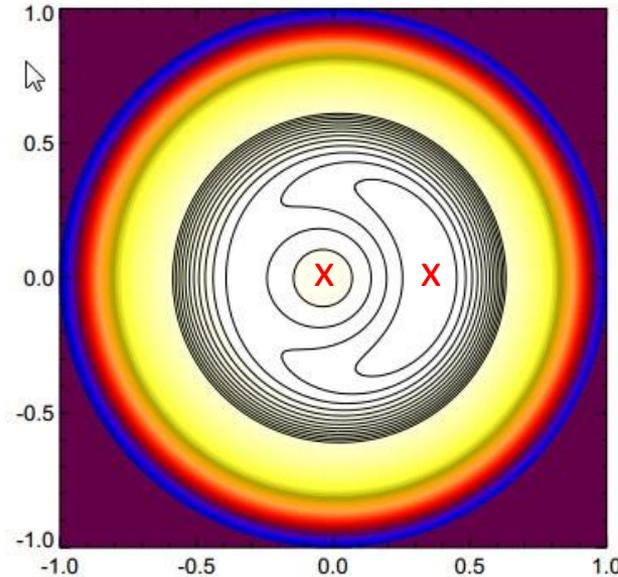
# Two-region (TR) relaxation model with current sheets

Island formation in Single Helical states changes the topology of the magnetic field

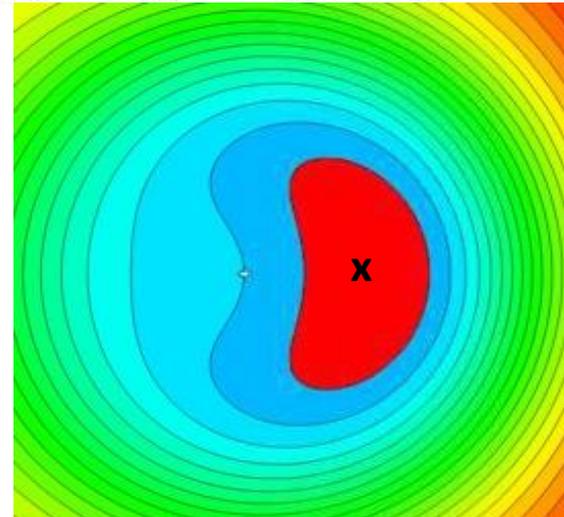
Two possible cases:

- i. **2 magnetic axes** corresponding to 3 topological regions
- ii. **1 magnetic axis** corresponding to 2 topological regions

**Case ii** corresponds to a **mode** saturated at **large amplitude** and can be treated within a **two-region relaxation model**



**2 magnetic axes**



**1 magnetic axis**

## TR relaxation model with current sheets (2)

- The presence of a reconnecting mode can create current sheets in the plasma
- We assume that in this two region a Taylor's like relaxation is going on
- We assume a given Taylor's like expansion for the fields in each region and match the solutions at the location of the current sheet :

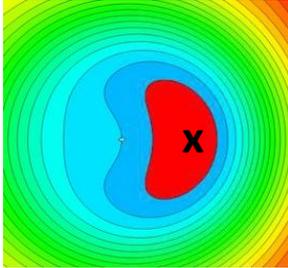
$$\begin{aligned} \mathbf{B} &= (0, J_1(\alpha_1 r), J_0(\alpha_1 r)) \quad \text{in} \quad 0 \leq r < r_c \\ \mathbf{B} &= (0, b_1 J_1(\alpha_2 r) + b_2 Y_1(\alpha_2 r), \\ &\quad c_1 J_0(\alpha_2 r) + c_2 Y_0(\alpha_2 r)) \quad \text{in} \quad r_c < r \leq a \end{aligned} \quad \longrightarrow \quad \nabla \times \mathbf{B} = \alpha(r) \mathbf{B}$$

## RELAXのSH RFP状態と緩和モデルとの比較検討

R. Paccagnella, S. Masamune, A. Sanpei, “Relaxation models for single helical reversed field pinch plasmas at low aspect ratio”, Phys. Plasmas 25, 072507 (2018)

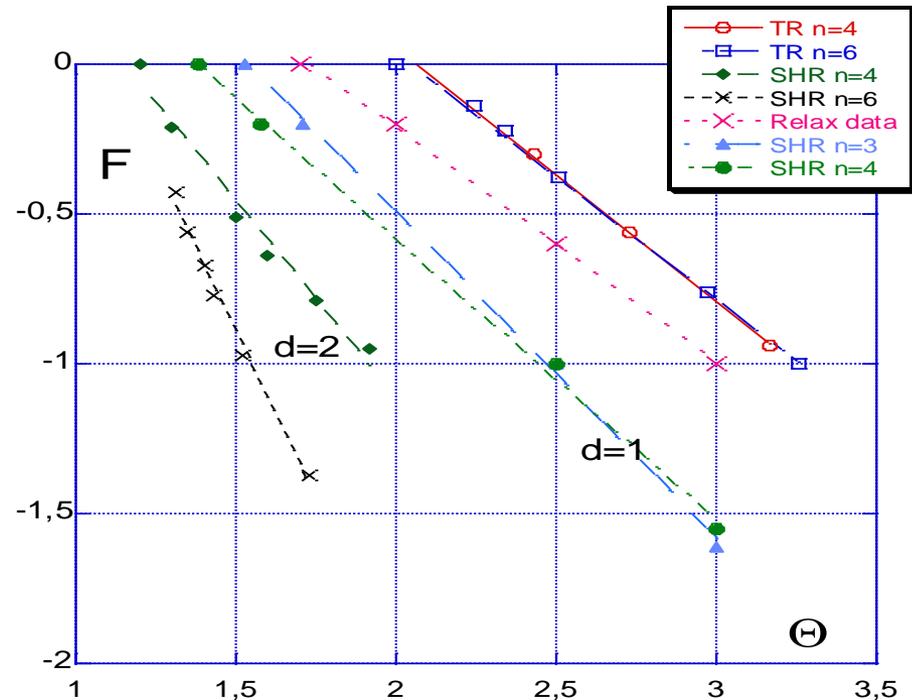
# Comparison of the experiments with the relaxation model predictions

TR model



SHR model

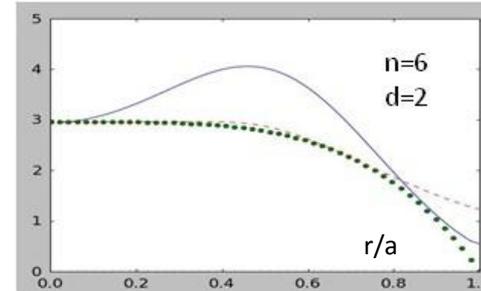
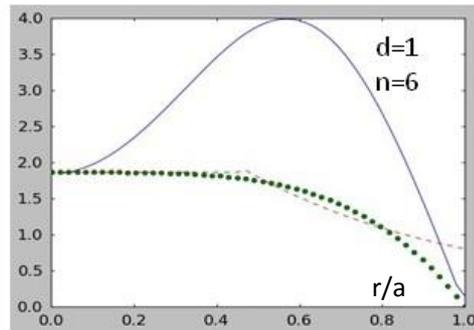
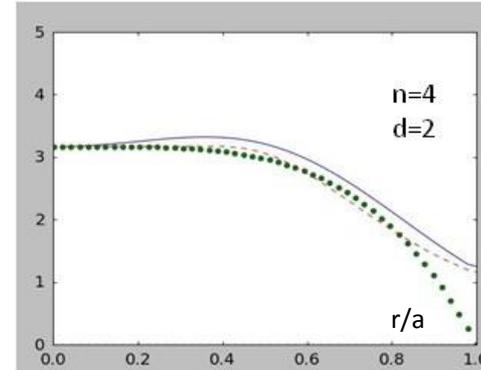
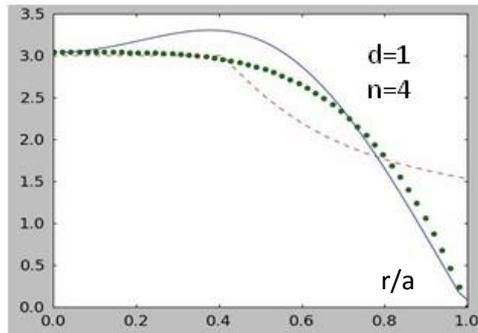
$$\mathbf{J} = \frac{\lambda_0(2+d)}{2} \chi^d \mathbf{B}$$



F vs.  $\Theta$  : experimental fit (crosses also labeled by exp); d=1 and d=2 SHR predictions with n=3 and n=4 and n=4 and n=6 respectively; the 2 region model predictions for n=4 and n=6.

- In the experiments the dominant modes are almost aligned along the same curve.
- The model curves are quite far from the experimental fit for the SHR predictions with d=2, differently from the medium aspect ratio case.

# Looking into the model predictions: importance in parallel current density profile



$\lambda$  profiles vs. normalized radius ( $r/a$ ).

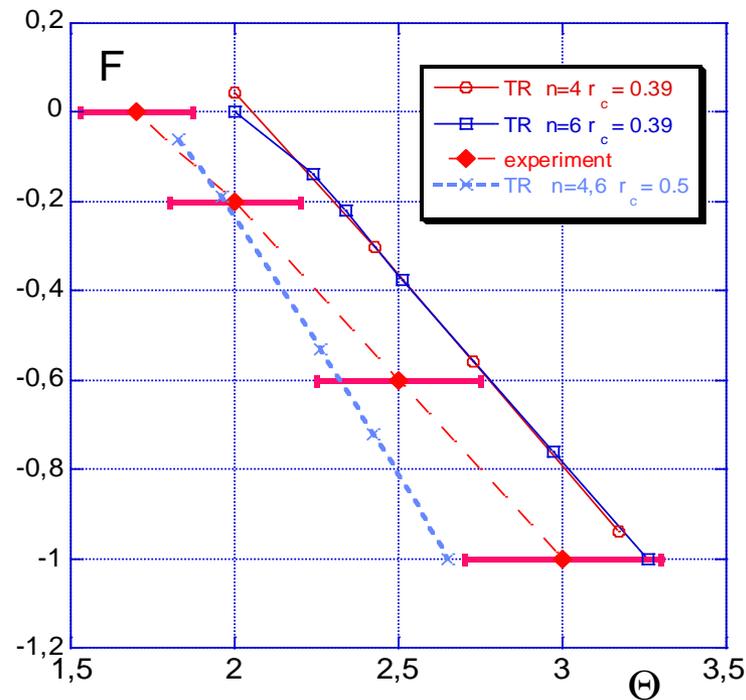
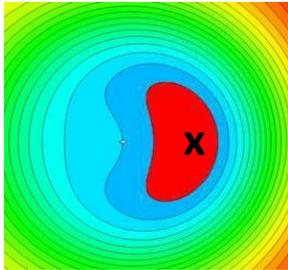
Dotted lines: experimental fit; plain lines: SHR; dashed lines: TR curves.

Top curves refer to shallow and bottom curves to deeper F cases.

- For the SHR model, there is a tendency to develop a bump in  $\lambda$  profile by decreasing the F value.
- Apart a small region near the edge, when  $d$  is set to 2, the TR model is very close to the experimental fit for both reversal ratios, with assumption that  $\lambda_0$  be equal to the average of the  $\lambda$  profile in the region from the maximum to the axis.

# Appropriate choice of parallel current density and resonant surface location improves the agreement between the experiment and TR model

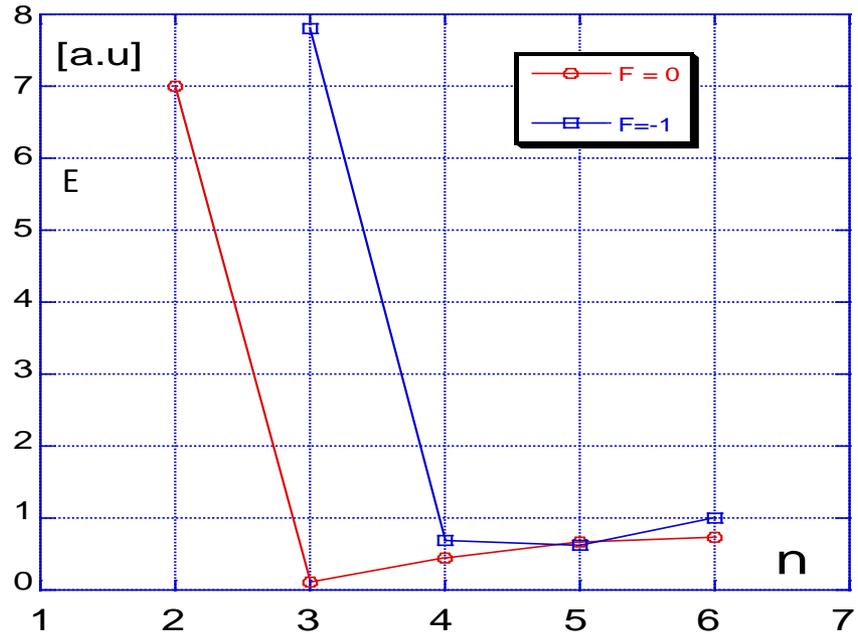
TR model



$F$  vs.  $\Theta$  : experimental fit with error bars (red long dashed), TR model with  $r_c=0.4$  (circles and squares), TR model with  $r_c=0.5$  (light blue short dashed).

- There is a good agreement between the TR model and the experimental data within their typical error bars.
- It is only by using the SHR and the TR model that we could obtain a good fit. The SH eigenvalue  $\lambda(0)$  and the position of the resonance of the dominant mode are the two essential ingredients used within the TR model.

## Predictive capability of the model: preferred n minimizes the integrated “error” E at different F’s



Integral error E vs. n for shallow and deep F values.

$$\text{Integrated error: } E(q_s) = \int \left( J_{\parallel}^{\lambda_{0,1}} - J_{\parallel}^{\chi} \right)^2 dr$$

$J_{\parallel}^{\lambda_{0,1}}$ : deduced by the TR model;  $J_{\parallel}^{\chi}$ : deduced directly from the SHR model

## Summary and Conclusions :

- The SH-relaxation model seems **NOT ABLE** to predict the experimental  $F-\Theta$  trend (including  $n=4$  and  $n=6$  alignment) at  $R/a=2$
- The two-region Taylor's model can fit the experimental data with an appropriate choice of the truncation radius, i.e. position of the current sheet and assuming a flat core  $\lambda$  as the **AVERAGE** obtained from the SH model
- **Dominant  $n$  modes at various  $F$  predicted correctly by the CLOSENESS between the two models (SH and 2 regions) of relaxation**

### Possible explanations:

- A residual stochasticity is flattening the  $\lambda$  profile in the core (the parallel current bump is not observed experimentally)
- Need for a toroidal theory? (but difficult) (the relaxed SH model is obtained assuming cylindrical geometry)

## Comments on the relation of the TR model with tokamak research:

### Importance of helical relaxed state in fusion plasmas

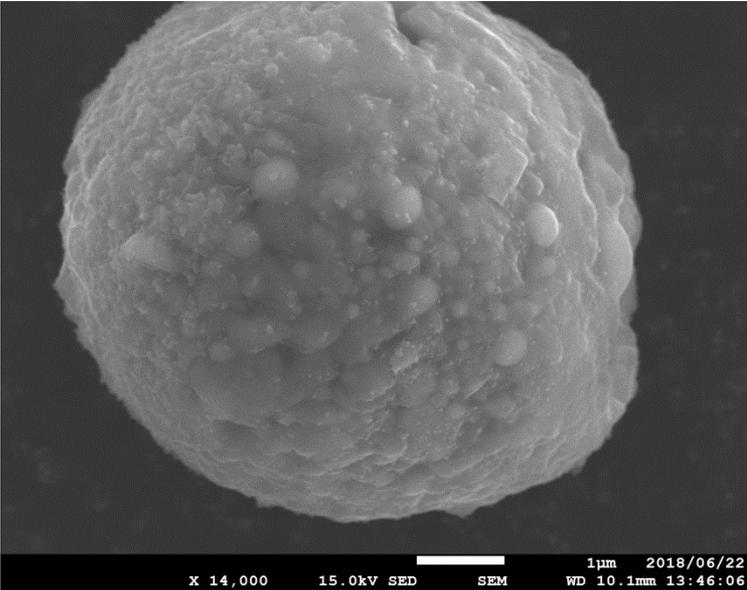
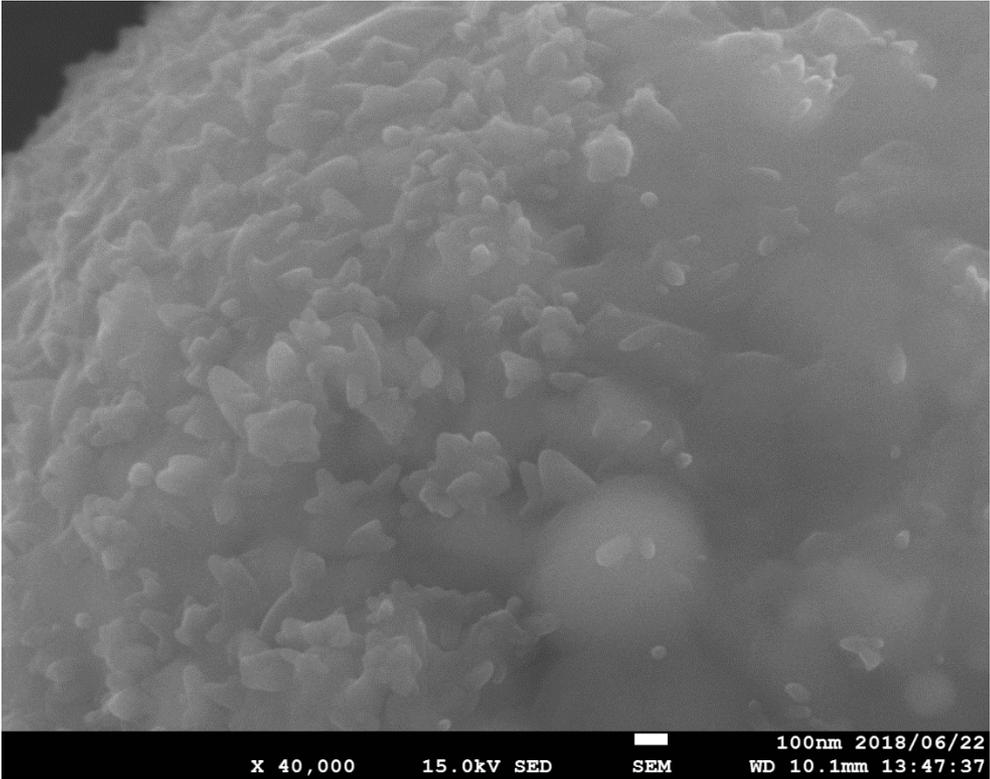
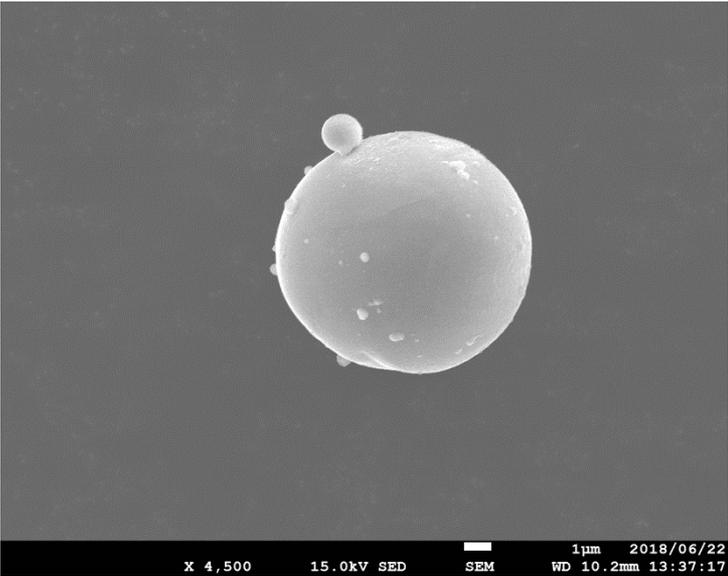
- **Hybrid operation scenario** for high-beta tokamak core
- Spontaneous formation of the helical core
- A possible explanation for the helical core is given by Piovesan et al.

“Impact of ideal MHD stability limits on high-beta hybrid operation”,  
Plasma Phys. Control. Fusion 59, 014027 (2017)

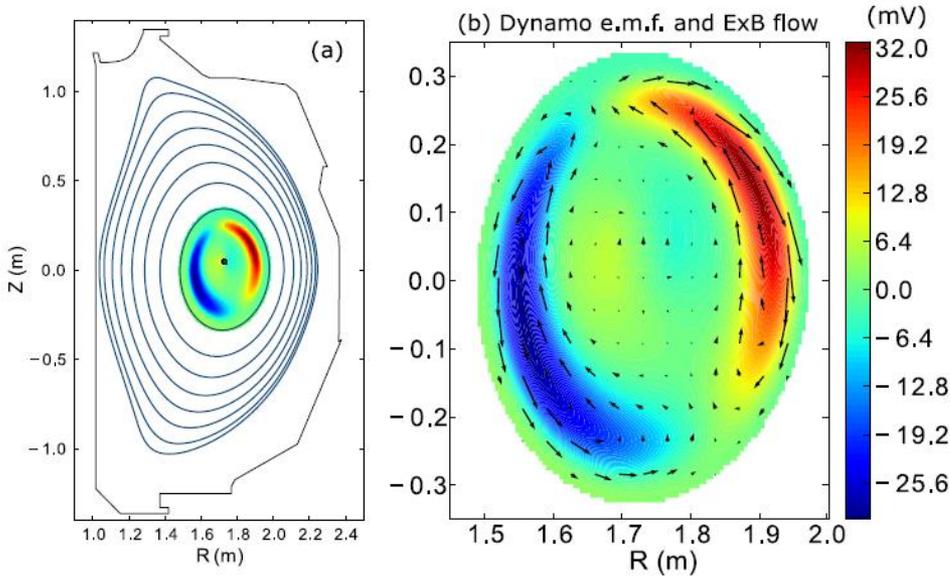
“Role of a continuous MHD dynamo in the formation of a 3D equilibria  
in fusion plasmas”, Nucl. Fusion 57, 076014 (2017)

where **similarities to the electrostatic dynamo for helical RFP** is emphasized

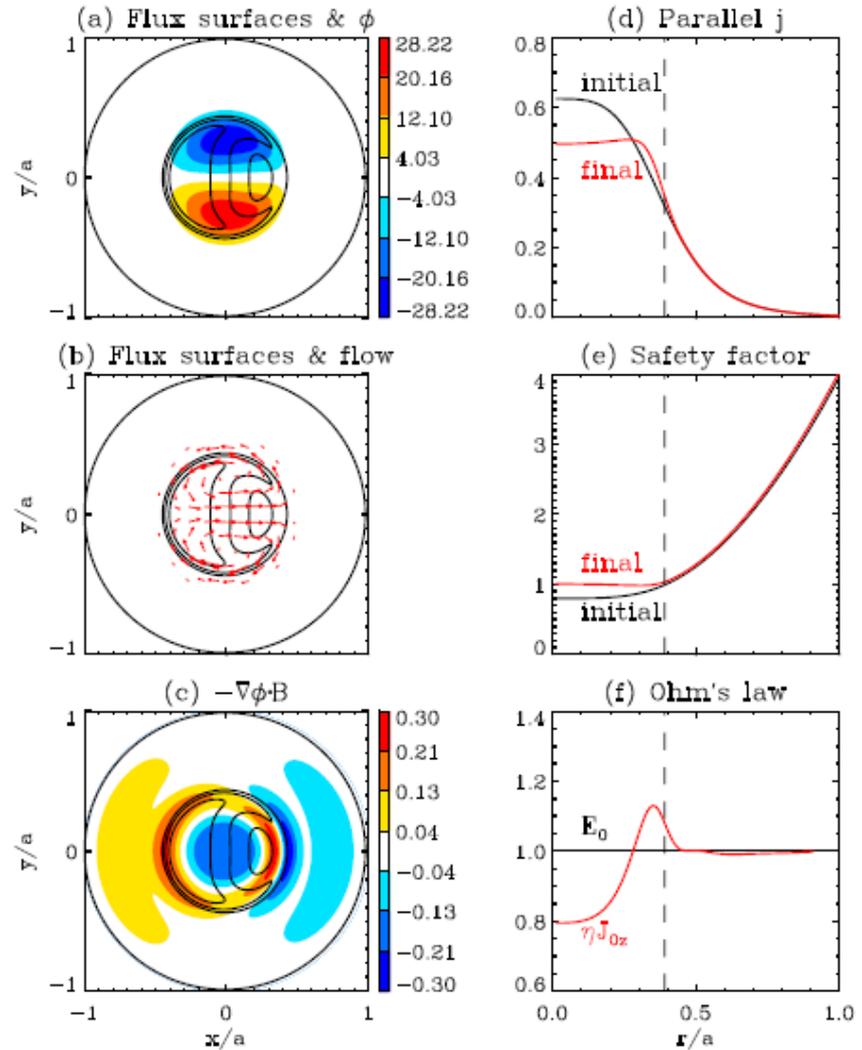
Appendix: Some examples of dust particles in the RELAX vacuum vessel





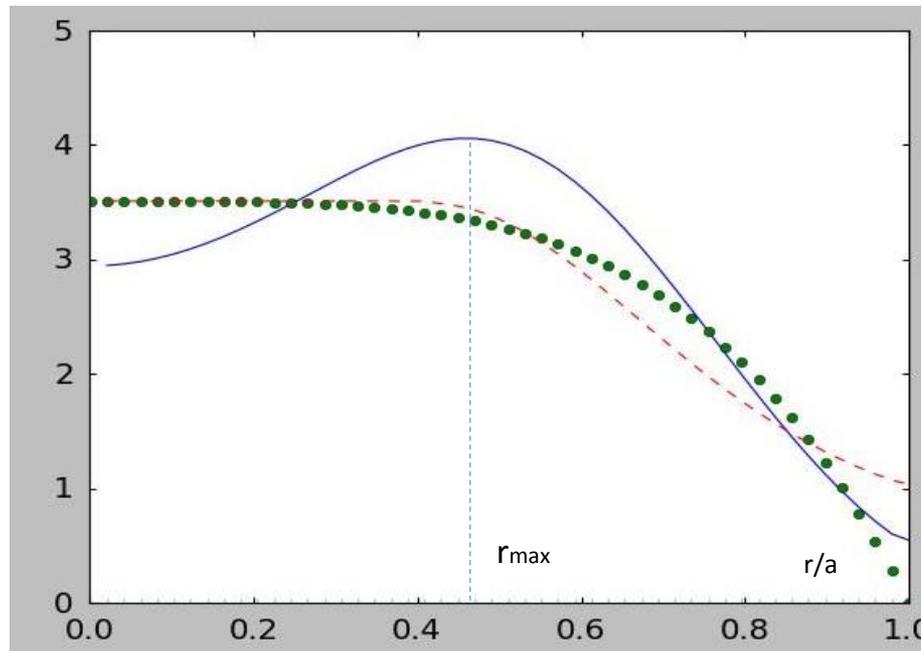


Helical flux surface reconstruction and electrostatic dynamo emf in DIII-D (#164661)

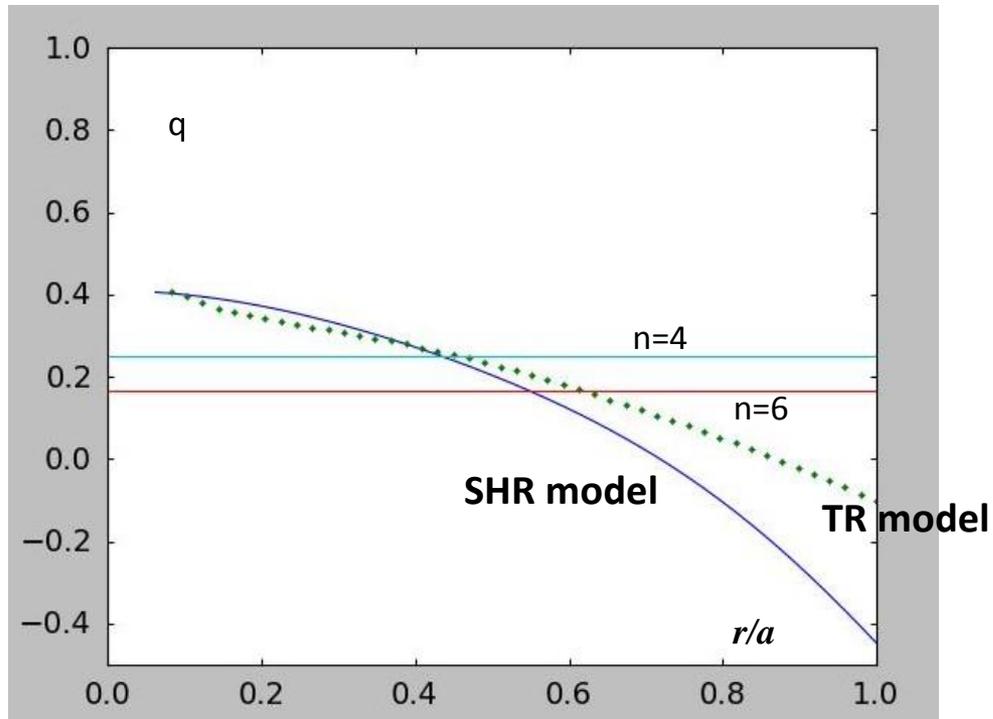


saturation of 1/1 mode leads to helical core in presence of external 1/1 field. (Specytl simulation)

The assumption that  $\lambda_0$  be equal to the average of the  $\lambda$  profile in the region from the maximum to the axis



$\lambda$  vs. normalized radius ( $r/a$ ): the dotted line is the experimental fit, the plain line the SHR model ( $d=2$ ) and the dashed line is the TR model. Note that the TR  $\lambda(0)$  is set to the average of the plain line (SHR model) between  $r_{\max}$  and  $r=0$ .



q profile vs. normalized radius ( $r/a$ ):  
 Plain line: SHR model; crosses: TR model  
 Resonances of  $n=4,6$  modes are shown.

Careful analysis of the matching radius in the TR model is important at low aspect ratio