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Trapping, anomalous transport

&

quasi-coherent structures

in magnetically confined plasmas

Madalina Vlad and Florin Spineanu

National Institute of Laser, Plasmas and Radiation Physics,

P. O. Box MG-36, Magurele, Bucharest, Romania

E-mail: madi@ifin.nipne.ro

The ExB stochastic drift in turbulent plasmas can determine

trajectory trapping or eddying

↓

<u>structures of trajectories</u>

<u>and non-standard statistics</u>

We present here

1) ***The statistical properties of test particle trajectories in the presence of trapping obtained with analytical methods,***

&

2) ***first results about test modes on turbulent plasma that take into account trajectory trapping.***

(universal drift turbulence with constant magnetic field and density gradient)

1) Test particle statistics in stochastic potential

We consider a turbulent plasma with given (measured) statistical characteristics of the stochastic potential and study test particle motion

Non-linear stochastic equation:

$$\boxed{\frac{d\vec{x}(t)}{dt} = \vec{v}(\vec{x}(t),t), \quad \vec{x}(0) = 0, \quad \vec{v}(\vec{x},t) = -\frac{\nabla\phi \times \vec{e}_z}{B}}$$

where $\vec{v}(\vec{x},t)$ is a stationary and homogeneous stochastic velocity field with Gaussian distribution and known spectrum or ***Eulerian correlation* (EC):**

$$E(\vec{x},t) = \langle \phi(\vec{x}_1,t_1)\phi(\vec{x}_1+\vec{x},t_1+t)\rangle = \Phi^2 f\left(\frac{\vec{x}}{\lambda_c},\frac{t}{\tau_c}\right),$$

where Φ is the amplitude , λ_c the correlation length, τ_c the correlation time.

The Kubo number : $K = \frac{V\tau_c}{\lambda_c} = \frac{\tau_c}{\tau_{fl}}, \quad \tau_{fl} = \frac{\lambda_c}{V}, \quad V = \frac{\Phi}{\lambda_c}$

(describes the ***decorrelation due to time variation*** of the stochastic field).

To determine: The statistical properties of the trajectories:

the probability of displacements (pdf) **P(x,t)**

$$\left\langle x^2(t)\right\rangle = \int P(x,t)\,x^2\,dx,$$

$$D(t) \equiv \frac{1}{2}\frac{d\left\langle x^2(t)\right\rangle}{dt}, \qquad D = \frac{\left\langle x^2(\tau_c)\right\rangle}{\tau_c}$$

***Lagrangian velocity correlation* (LVC) :** $L_{ij}(t) \equiv \left\langle v_i(0,0)\,v_j(\vec{x}(t),t)\right\rangle$

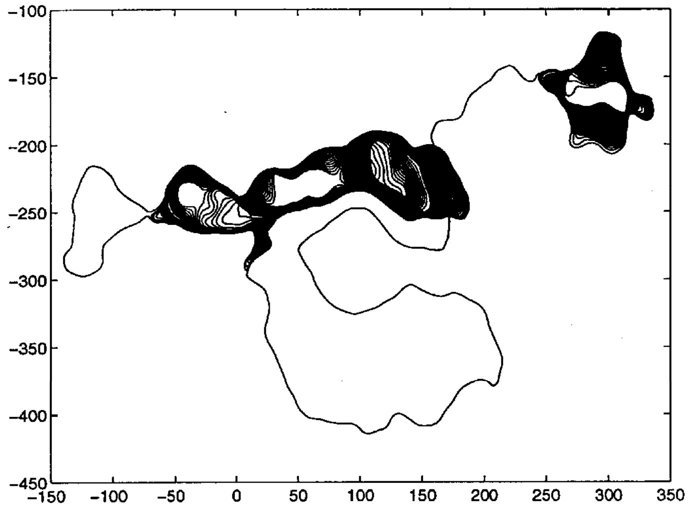
$$\left\langle x^2(t)\right\rangle = 2\int_0^t (t-t')L_{xx}(t')\,dt', \qquad D(t) = \int_0^t L_{xx}(t')\,dt',$$

Note:

-Test particle diffusion coefficients are the same with those obtained from the correlation of velocity and density fluctuation if there is space-time scale separation between fluctuations and the main gradients (due to the zero divergence of the ExB drift);

- The characteristics of the turbulence and all the other components of the motion are input data

There are two strong constraints for the statistical methods:



(A) The invariance of the potential for the static case

(Hamiltonian equation of motion)

$$\frac{d\phi(\vec{x}(t),t)}{dt} = \cancel{v_i(\vec{x}(t),t)\frac{\partial\phi(\vec{x}(t),t)}{\partial x_i}} + \frac{\partial\phi(\vec{x}(t),t)}{\partial t} = \frac{\partial\phi(\vec{x}(t),t)}{\partial t}$$

- ***static case*** ($K = \infty$): invariance of the potential and ***permanent trapping*** on the contour lines;
- ***slowly varying potential*** (**K > 1**): approx. invariance of the potential and ***temporary trapping***
- potential with ***fast time variation*** (**K < 1**): no trapping.

(B) The *statistical invariance* of the Lagrangian velocity due to $\nabla \cdot \vec{v}(\vec{x},t) = 0$

$$P[\vec{v}(\vec{x}(t),t)] = P[\vec{v}(\vec{x}(0),0)] = P[\vec{v}(\vec{x},t)]$$

for stationary and homogeneous turbulence.

Semi-analytical statistical methods

The existing analytical methods
(Corrsin Approximation, DIA, functional integration) are not compatible with the conditions (A) and (B). They do not describe trapping and lead to diffusive transport in in the static potential

❑ *The decorrelation trajectory method* ***(DTM)*** *1998*

(M.Vlad, F. Spineanu, J. H. Misguich, R. Balescu, "Diffusion with intrinsic trapping in 2-d incompressible stochastic velocity fields", **Physical Review E 58** (1998) 7359)

- DTM is based on a set of simple (deterministic) trajectories determined from the Eulerian correlation EC of the stochastic potential.
- main consequences of condition (A) are fulfilled, condition (B) is not;
- ***Main physical result***: ***trapping produces memory effects*** ***(long-time correlation of the Lagrangian velocity)*** and subdiffusive transport for static potential.

❑ ***The nested subensemble method (NSM)*** *2004*

(M. Vlad, F. Spineanu, “Trajectory structures and transport”, **Physical Review E 70** (2004) 056304(14))

- NSM is a systematic expansion based on dividing the space of realizations of the stochastic potential in nested subensembles. NSM yields detailed statistical information.
- all consequences of (A) are fulfilled, condition (B) is improved (but not enough in order 2)
- ***Main physical result***: ***trapping determines coherence*** in the stochastic motion and ***quasi-coherent trajectory structures***.

❑ ***The velocity on structure method (VS)*** *2008*

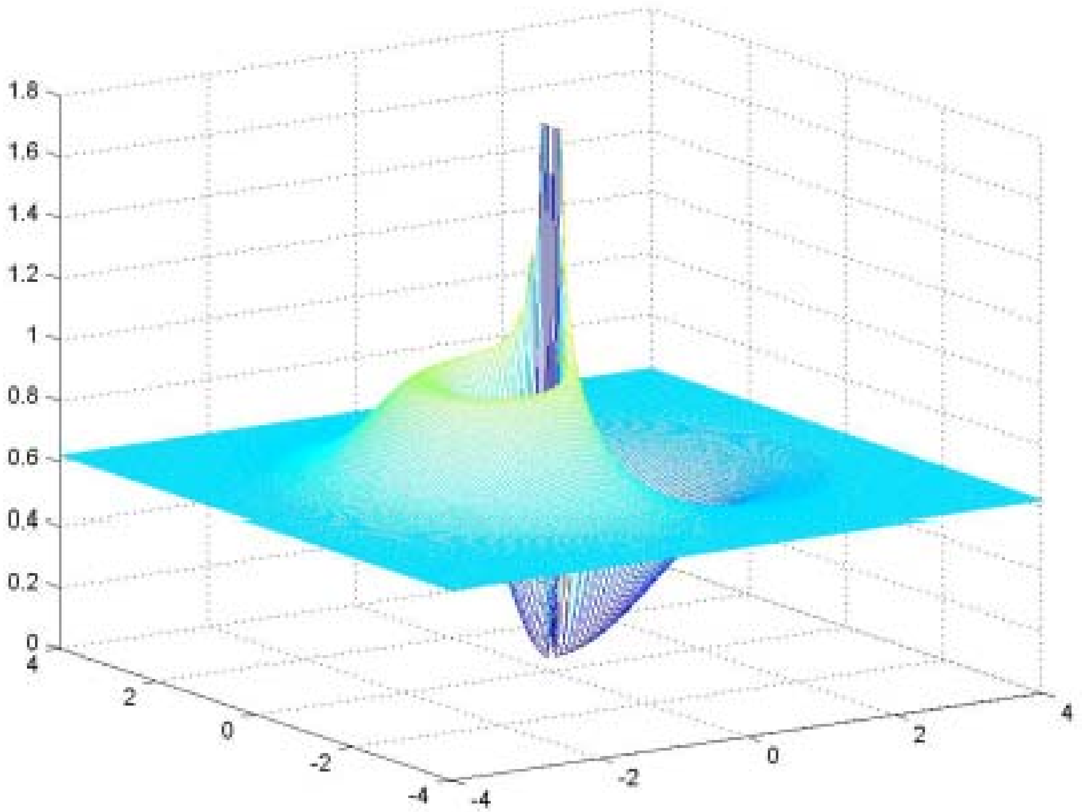
- ***Both conditions (A) and (B) are respected***;
- Trajectory structures and the memory effects are confirmed and better described
- ***Main physical result***: ***the distribution of displacements until decorrelation is determined*** (the 1-step pdf).

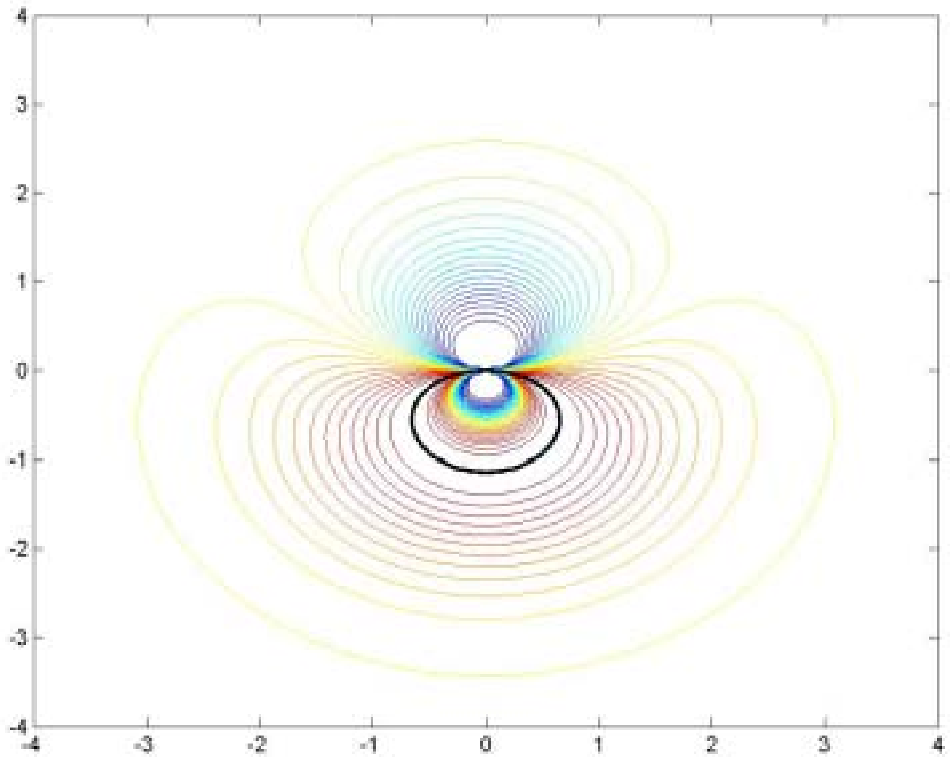
***It is strongly non-Gaussian in the trapping regime* (K>>1)**.

➢ ***The main ideas of the VS:***

- the contour lines of the potential with the same value ϕ form ***geometrical structures***.

The (conditional) probability that the potential in a point x is ϕ, given that it has the same value in x=0 and a fixed orientation of the gradient:





The maximum of the probability is on the contour line ϕ of the average potential; The size of the structure increases when ϕ decreases.

- the Eulerian velocity on the contour lines with the same ϕ has the same probability as in the whole set o realizations $\longrightarrow$ the characteristics of the 1-dimensional motion are the same for all structures.

- the motion along the structure (along the contour line of the average potential) is a ***"standard" stochastic motion***.

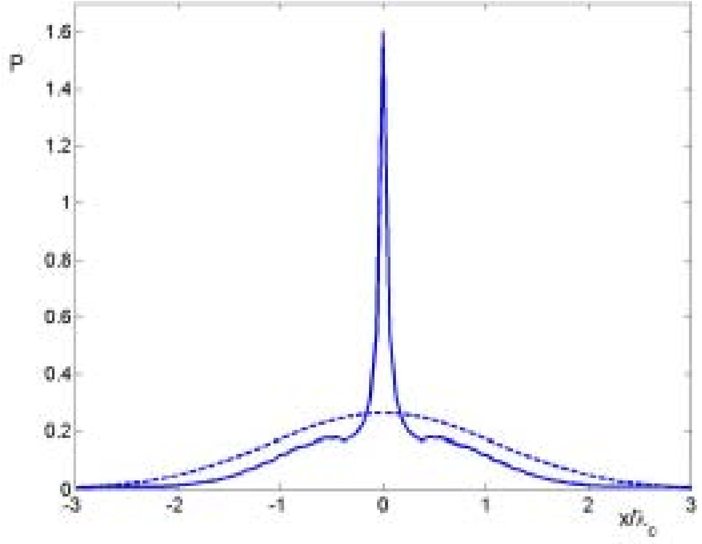
-This motion eventually leads to *uniform distribution of the particles on the structure*. Thus, the ***geometrical contour line structures*** become ***trajectory structures*** in a time τ_S

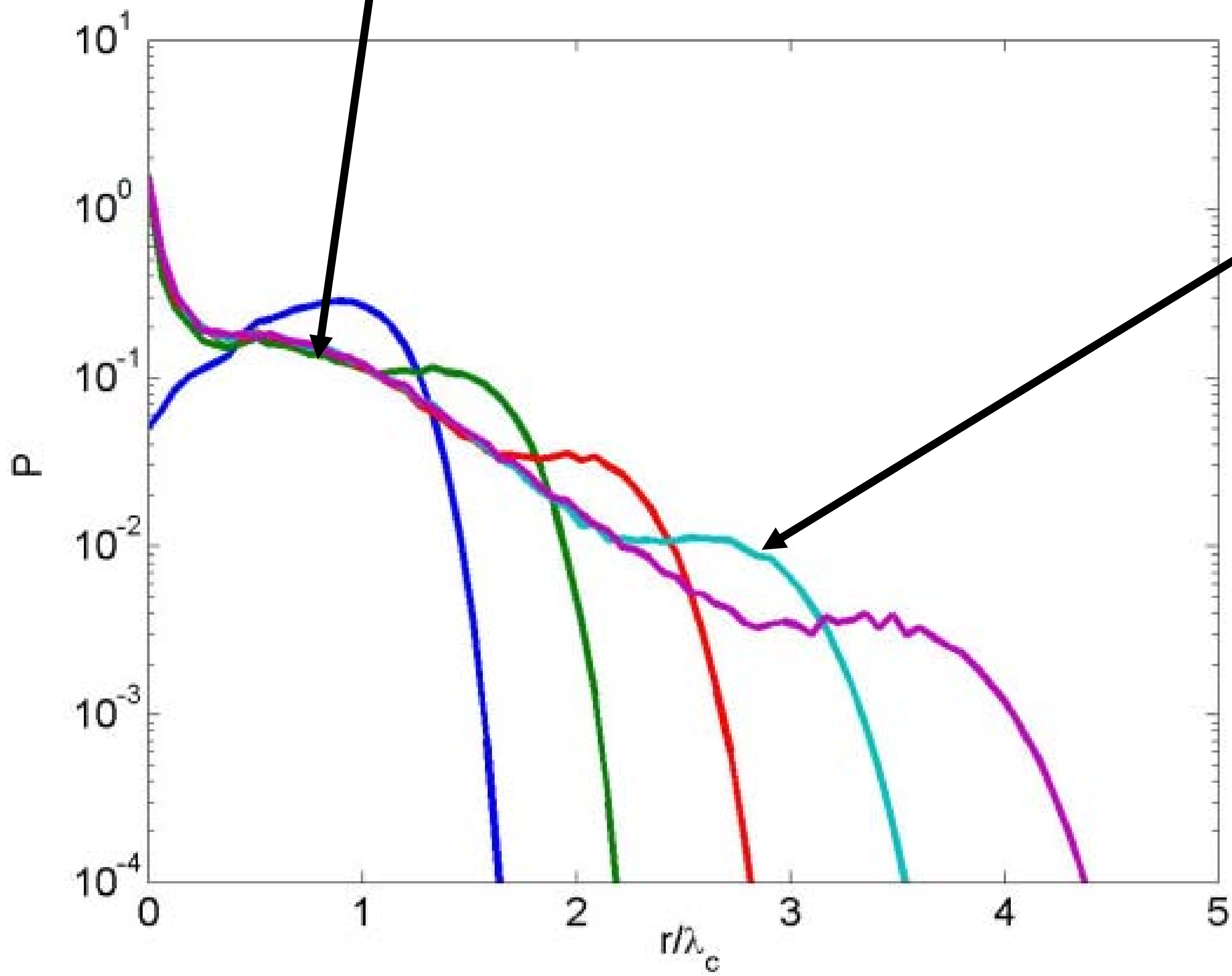
τ_S is a decreasing function of ϕ . Thus the formation time of a trajectory structure increases when the size of the structure increases.

➢ The pdf of the displacements is obtained by summing the contribution of all values of ϕ (by the superposition of all trajectory structures)

The probability of displacements for static potential

Time invariant part formed by trajectory structures of different scales (vortical motion)





Moving maximum
of trajectories that
are not in a structure at that time (radial motion).
Decreasing number of free particles.

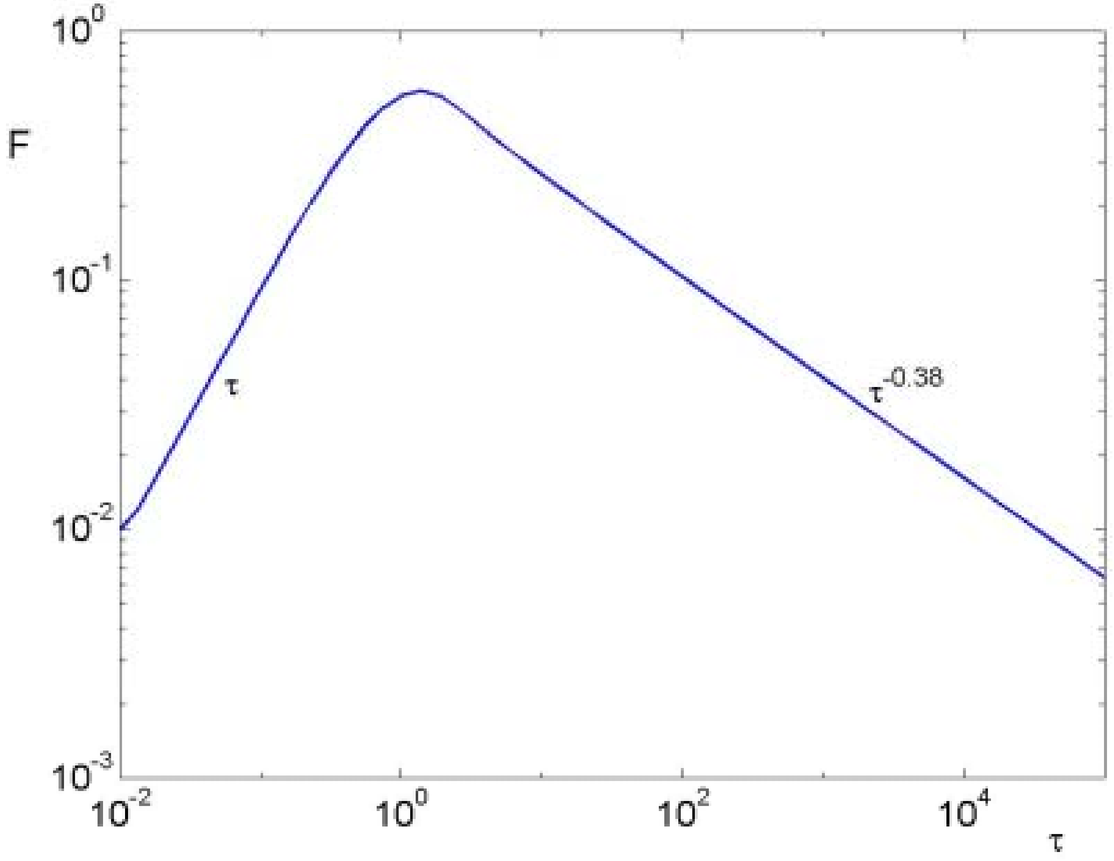
<u>Note:</u> this gives the probability of one-step jumps (fractal diffusion)

The diffusion coefficient for static potential:

$$D(t) \cong \frac{\left\langle x^2(t) \right\rangle}{t} = D_B F(t), \quad \left\langle x^2(t) \right\rangle = \int [P_{tr}(x) + P_f(x,t)] x^2 dx$$

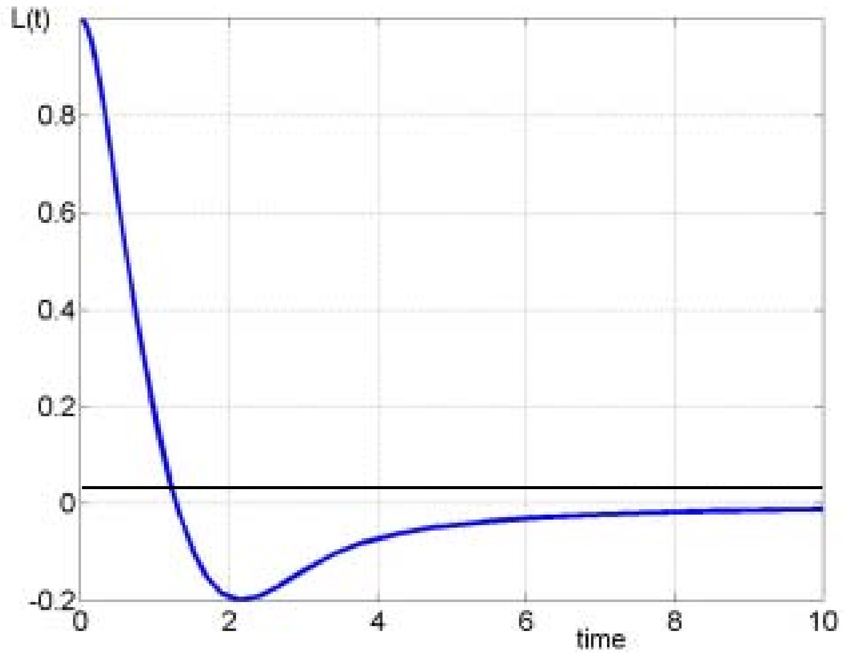
constant at large t

$\approx n_f(t) V^2 t^2$



Correlation of the Lagrangian velocity with long negative tail that exactly compensates the positive part at small time delay

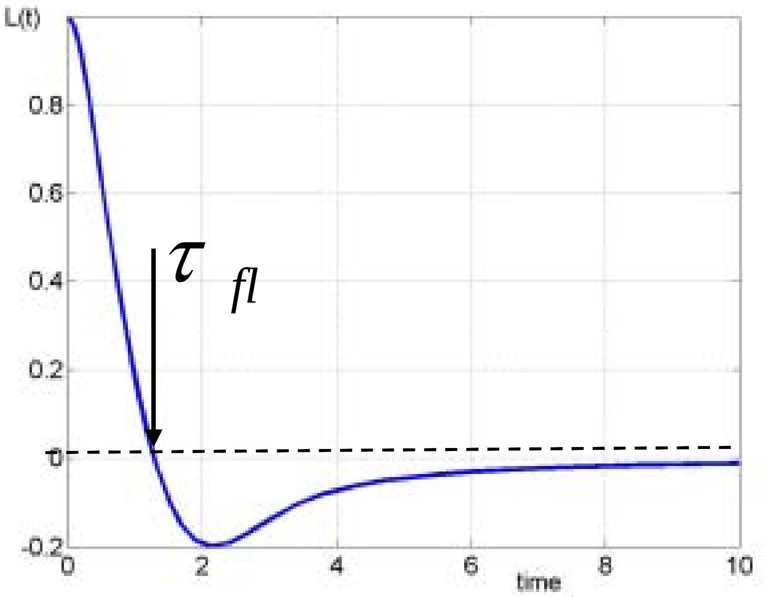
Decay of D(t) due to trapping



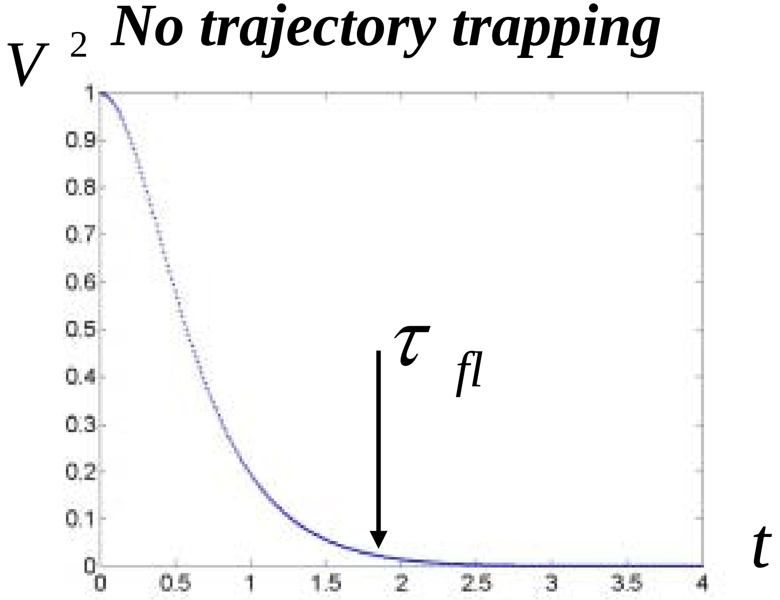
→ ***Subdiffusive transport*** ***(and memory !)***

Effect of a perturbation = decorrelation mechanism characterized by a time τ_d

Trajectory trapping



No trajectory trapping



- ***Strong decorrelation mechanism with small decorrelation time*** $\tau_d < \tau_{fl}$

$$D \downarrow \;\; when \;\; \tau_d \downarrow, \quad D \approx V^2 \tau_d$$

- ***Weak decorrelation mechanism with large decorrelation time*** $\tau_d > \tau_{fl}$

* The negative part of the LVC is cut out;
Anomalous diffusion regime (increased diffusion at stronger decorrelation)

$$D \uparrow \;\; when \;\; \tau_d \downarrow$$

* The LVC is not influenced;
Transport coefficient independent on τ_d and stable to such perturbations.

➢ **The effect of *time variation of the potential* (finite K):**

$$D(K) = D_B F(K), \quad D_B = \frac{\Phi}{B}$$

Decay of D due to trapping.

➢**The effect of parallel motion:** the correlation of the potential decays because particles move out of the correlated zone along the magnetic field.

Effective Kubo number: $$K_{eff} = \frac{K K_{II}}{K + K_{II}}, \quad K_{II} = \frac{\tau_{II}}{\tau_{fl}}, \quad \tau_{fl} = \frac{\lambda_{II}}{v_{II}}$$

$$K_{II} << 1 \rightarrow K_{eff} << 1 \longrightarrow$$ No trapping for electrons

➢ **The effect of collisions:** diffusion of the potential correlation.
E(x) is not destroyed but spreaded due to collisional motion.

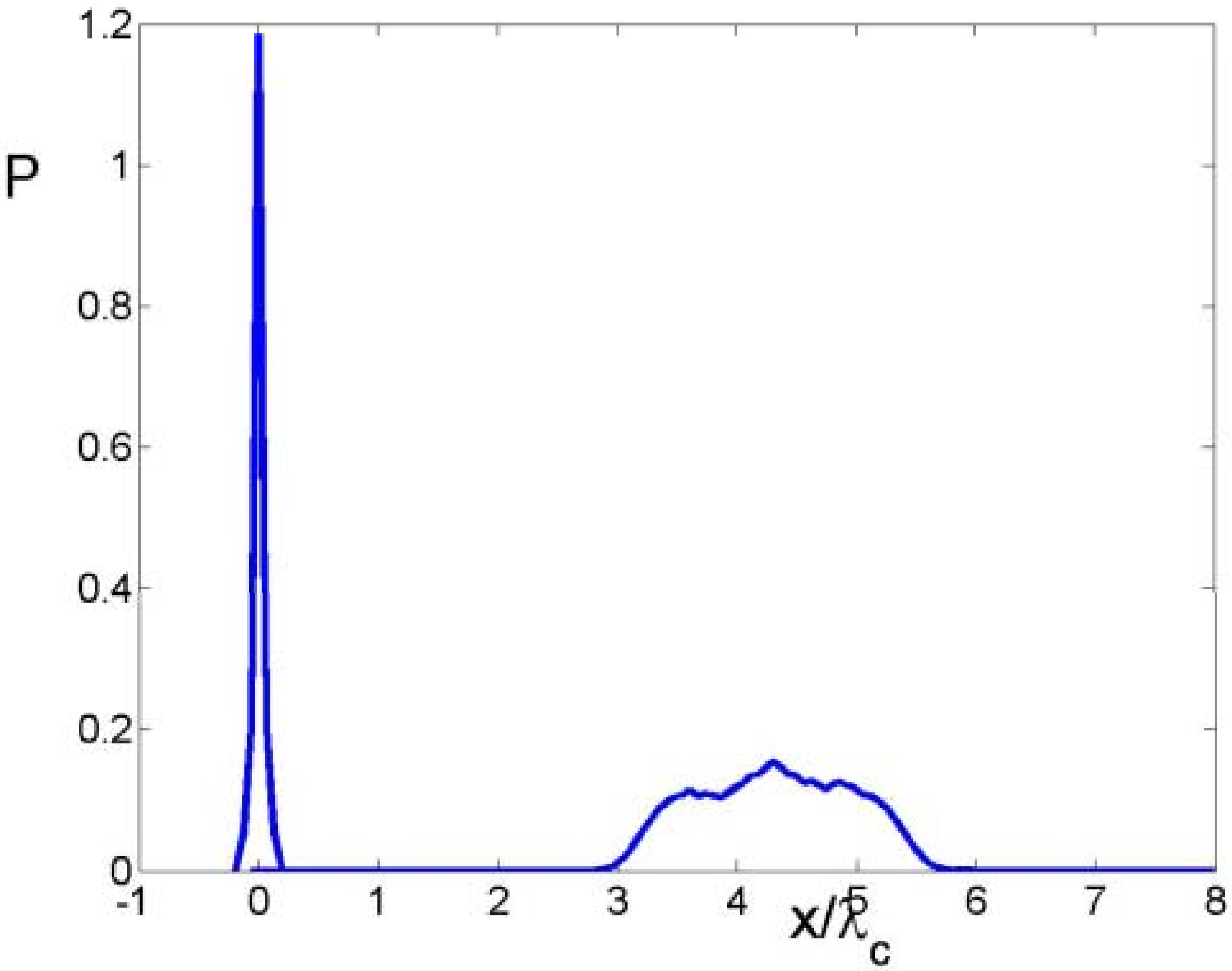
Effective Kubo number: $$K_{eff} = \frac{K}{(1 + 2\chi K)^{3/2}}, \quad \chi = \frac{D_0}{\beta}$$

Always increased diffusion for stronger decorrelation in the presence of trapping

➢ The effect of an average velocity: $\overline{V_d} = \frac{V_d}{V}$

- equivalent with a biased potential $\longrightarrow$ strips of open contour lines appear

For $\overline{V_d} > 1$ all the contour lines are opened and there is no trapping.



Trapping exists for $\overline{V_d} < 1$ and determines ***"the acceleration"*** of the free particles (on the opened contour lines), which reach an average velocity $V_f > V_d$

(not a dynamical effect but determined by the selection of the free particles)

- V_f exactly compensates the trapped particles:

$$\langle v(x(t)) \rangle = n_f V_f = V_d$$

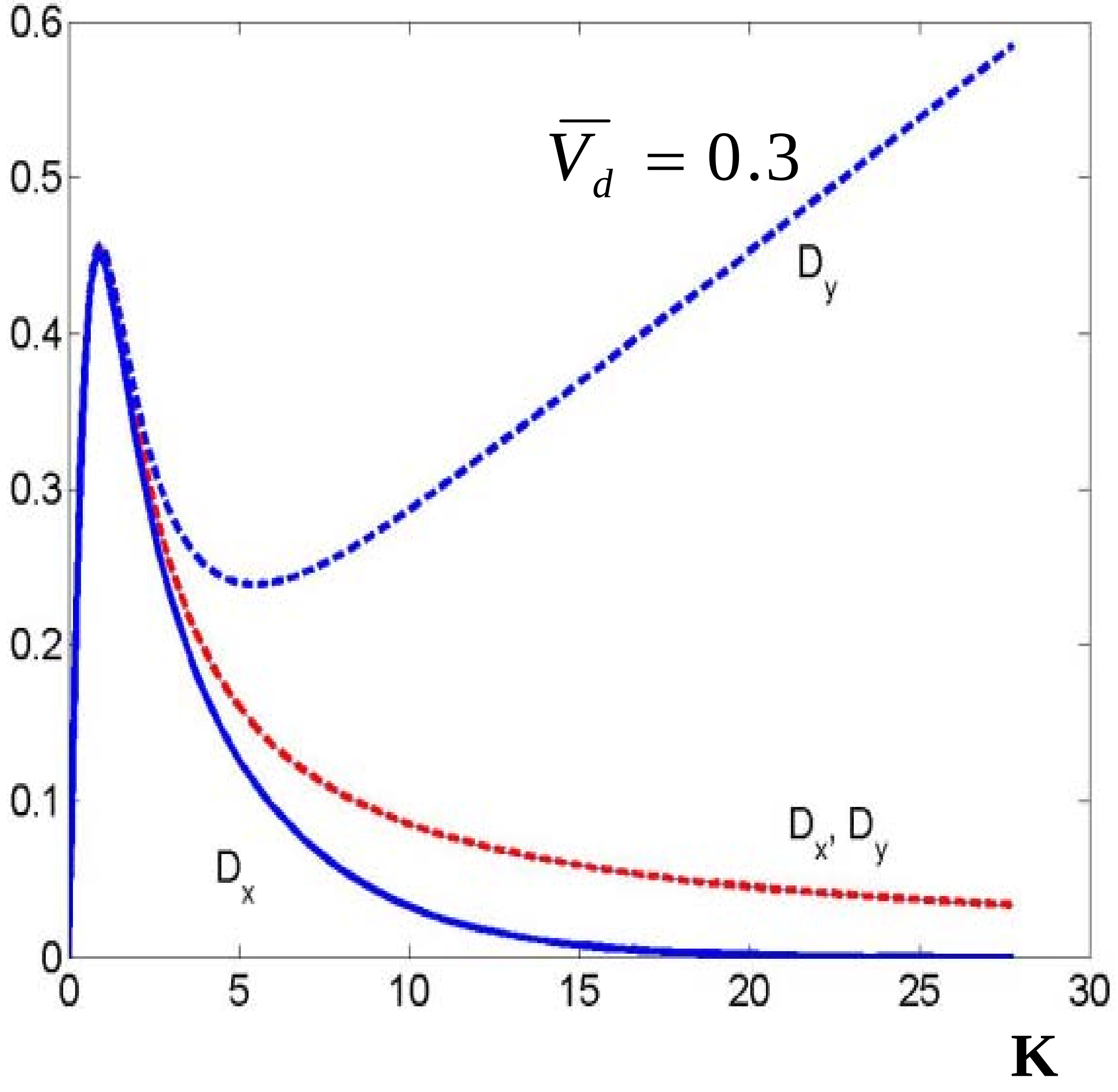
(statistical invariance of the Lagrangian velocity)

Very large amplification of the diffusion along the average velocity:

- ballistic transport for static potential

- D ~ K at large K

Strong decrease of the diffusion in the perpendicular direction ***at large* K**



The general conclusion of all this studies of test particles stochastic fields:

Trapping **determines:**

memory effects,

anomalous transport,

non-Gaussian distribution,

high degree of coherence

Trapping exists when K > 1 and if the other components of the motion are weak perturbations (collisions, average flow, …)

$$\chi = D_0 / V\lambda_c < 1, \quad V_d / V < 1, \ldots$$

2) Test modes in turbulent plasma

• We consider a turbulent plasma with given (measured) statistical characteristics of the stochastic potential and study linear test modes

• ***AIM: to find the effect of trajectory trapping on test modes***

The growth rate and frequency of the test modes are determined as function of the statistical characteristics of the background turbulence

The drift (universal) instability in cartezian slab geometry, with constant magnetic field and density gradient. Turbulence is developed, with inverse cascade and vortical structures.

x

The formal solution of the Vlasov equation for an initial perturbation with

$\varphi_0(\vec{x}, z)$ **obtained with the characteristic method (integration along trajectories):**

$$n^e(\vec{x},z,t) = n_0(x)\left[1 + \frac{e\phi(\vec{x},z,t)}{T} - \frac{e}{T}\int dv_z F_M(v_z)\int_0^t d\tau\left(\partial_\tau - V_*\partial_y\right)\phi\left(\vec{x}^e(\tau), z - v_z(t-\tau), \tau\right)\right]$$

$$n^i(\vec{x},z,t) = n_0(x)\left[1 + \frac{e\phi(\vec{x},z,t)}{T}\right] +$$

$$n_0(x)\frac{e}{T}\int d^2v\, F_M(v)\int_0^t d\tau \left\{V_*\partial_y\overline{\phi}\left(\vec{x} - \vec{\rho} + \delta\vec{\zeta}(\tau), z, \tau\right) + \right.$$

$$\left.\left(\frac{\varepsilon_{ij}}{B}\partial_j\overline{\phi}\left(\vec{x} - \vec{\rho} + \delta\vec{\zeta}(\tau), z, \tau\right)\partial_i - \partial_t\right)\phi\left(\vec{x} + \delta\vec{\zeta}(\tau), z, \tau\right)\right\}$$

$$n^e(\vec{x},z,t) = n^i(\vec{x},z,t) \longrightarrow \varphi(\vec{x},z,t)$$

- **the solution in the zero Larmor radius limit:**

$$\int_0^t d\tau \left(V_* \partial_y - \partial_t\right)\phi\left(\vec{x} + \delta\vec{\zeta}(\tau), z, \tau\right) \longrightarrow \varphi(\vec{x}, z, t) = \varphi_0(\vec{x} + V_* t, z)$$

The time evolution of the potential is its displacement with the diamagnetic velocity $\longrightarrow$ Kubo number in drift type turbulence is large (even in quasilinear case)
The existence of trapping is determined by the amplitude of the ExB drift: it appears when V is larger than the diamagnetic velocity.

$$V_d = V_* / V$$ ***Trapping parameter for drift type turbulence***

QL: $$\tau_{II}^{e} << \tau_* < \tau_{fl} << \tau_c < \tau_{II}^{i}$$

NL: $$\tau_{II}^{e} << \tau_{fl} < \tau_* << \tau_{II}^{i} < \tau_c$$

$$\tau_* = \lambda_c / V_* = 2\pi / \omega_*$$

Thus:

- ***trapping and the nonlinear effects are important for drift type turbulence;***
- ***nonlinear effects appear if*** $V_d < 1$ ***for ions***
- ***electrons have quasiliniar statistics***

Test modes on ***turbulent state of the plasma*** with a potential that has known EC $\phi(\vec{x},z,t)$

• The system is perturbed with a small potential $\delta\phi(\vec{x},z,t)=\delta\phi_{k\omega}\exp\left(i\vec{k}\cdot\vec{x}+ik_z z-i\omega t\right)<<\phi,$

which is small enough such that the trajectories are not much perturbed.

• the Vlasov (drift kinetic) equation for the response at the perturbation is linearized

$$\partial_t g^{\alpha}-\frac{\nabla\phi\times\vec{e}_z}{B_0}\cdot\nabla g^{\alpha}+v_z\partial_z g^{\alpha}=\frac{\nabla\delta\phi\times\vec{e}_z}{B_0}\cdot\nabla\left(n_0F_M^{\alpha}+h^{\alpha}\right)+\frac{e_\alpha n_0F_M^{\alpha}}{T_\alpha}\partial_t\delta\phi$$

• The solution is obtained as integral along the trajectories determined by the unperturbed potential

$$g^{\alpha}(\vec{x},z,\vec{v},t)=\frac{e_\alpha n_0F_M^{\alpha}}{T_\alpha}\int_{-\infty}^{t}d\tau\left[\partial_t-\tilde{\vec{V}}_{*\alpha}\cdot\nabla\right]\delta\phi\left(\vec{x}^{\alpha}(\tau),z^{\alpha}(\tau),\tau\right)$$

Fluctuation of the diamagnetic velocity
determined by the background turbulence

$$\tilde{\vec{V}}_{*\alpha}=\frac{T_\alpha}{e_\alpha n_0B_0}\vec{e}_z\times\nabla(n_0+\bar{h}^{\alpha})$$

$$\frac{d\vec{x}^{\alpha}}{d\tau} = -\frac{\nabla\phi(\vec{x}+\vec{V}_{*}t)\times\vec{e}_{z}}{B_{0}}, \quad \frac{dz^{\alpha}}{d\tau} = v_{z}^{\alpha}$$

$$\vec{x}^{\alpha}\left(t;\vec{x},z\right) = \vec{x}, \quad z^{\alpha}\left(t;\vec{x},z\right) = z$$

The background turbulence produces the ***stochastic drift in test particle trajectories***,

- the characteristic times ordering is used for simplifications: the parallel motion of the ions and the ExB drift for electrons are neglected

❑ ***the renormalized propagator that takes into account trajectory trapping***

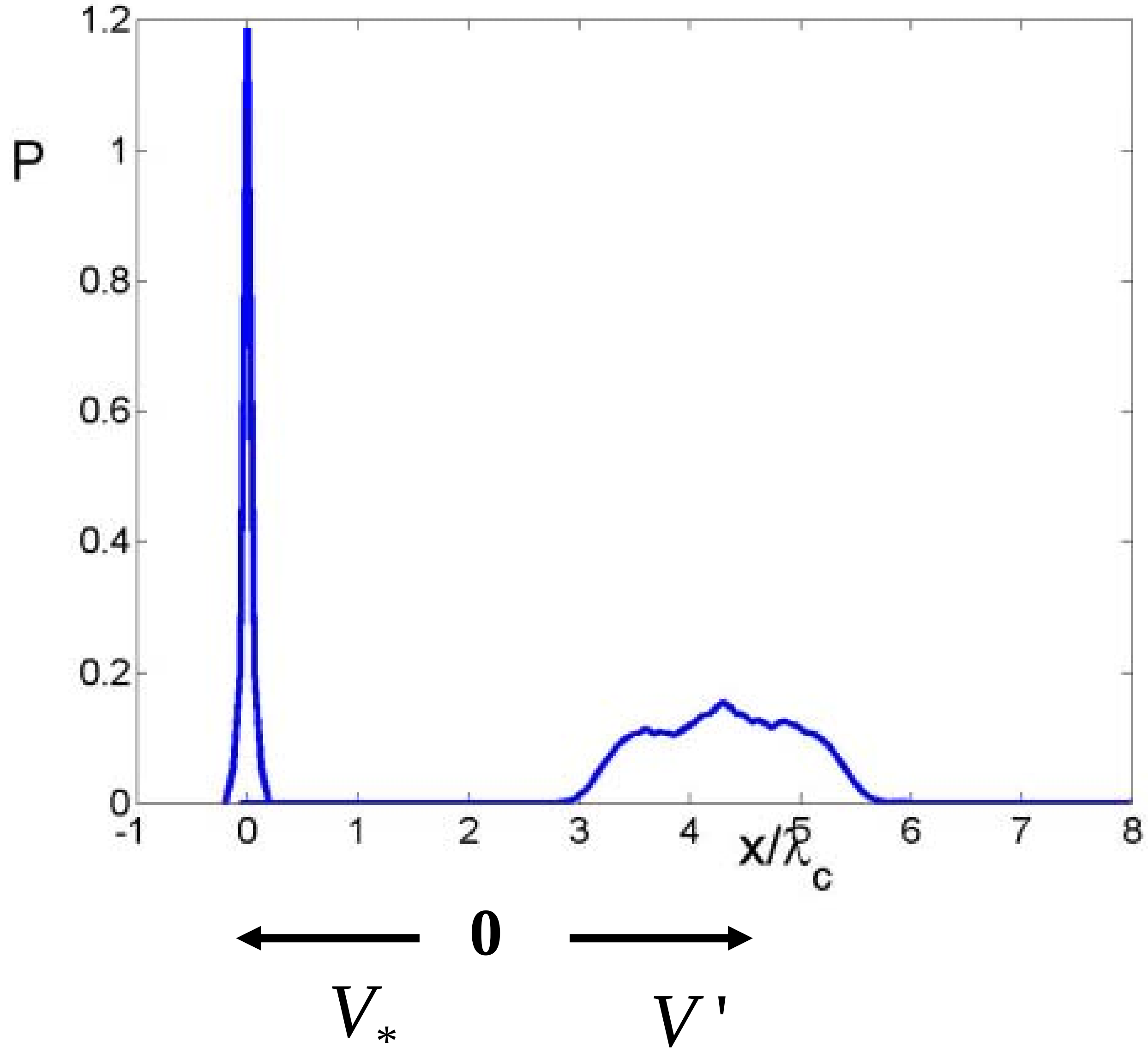
Effects of potential drift with the diamagnetic velocity and of trajectory structures appear

$$\Pi \cong i\exp(-k_{i}{}^{2}S_{i})\left[\frac{n_{tr}}{\omega+k_{y}V_{*}} + \frac{n_{f}}{\omega+k_{y}V'+ik_{i}{}^{2}D_{i}}\right]$$

The size of
trajectory structures

A moving stochastic potential is equivalent (up to a reference system change) with a biased potential.

Trapped particles move with V_* and free particles move in opposite direction



$$\langle x(t) \rangle = 0$$

$$\langle x(t) \rangle = n_{tr} V_* t + n_f V' t = 0$$

$$V' = -V_* \frac{n_t}{n_f}$$

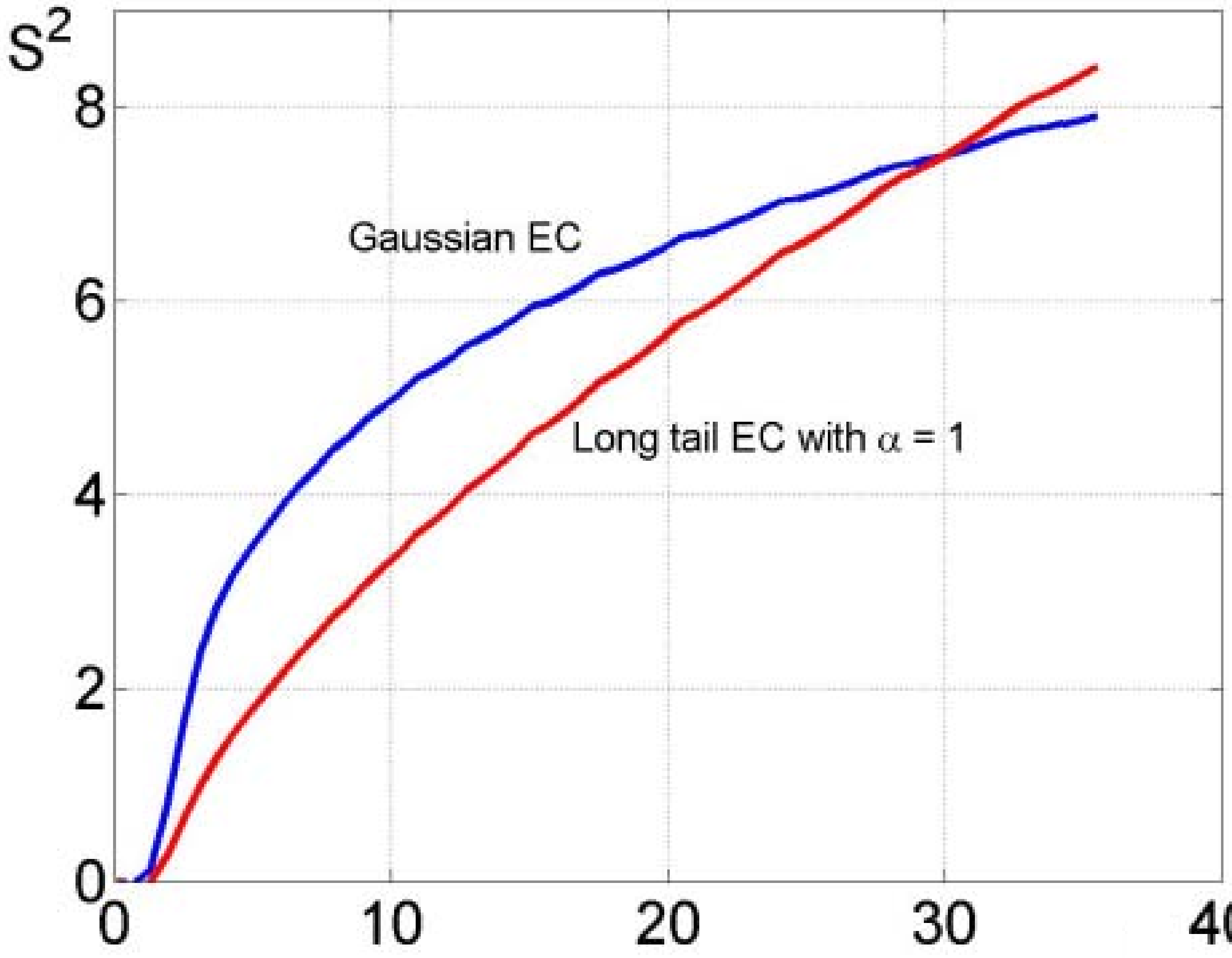
$\langle x^2(t) \rangle$ is large and has ballistical time-dependence (because of this separation)

The potential with slow motion $\overline{V}_* < 1$

destroys the large structures of trajectories, but smaller ones still exist.

The average size of the trapped trajectories is zero for $\overline{V}_* > 1$
and increases with the increase of $\overline{V}_*^{-1}$***, as a power law***

$S^2(\overline{V}_*) \approx \frac{1}{\overline{V}_*^{\nu}}$ for $\overline{V}_* < 1$, where ν depends on the EC $(\nu = 0.37, \quad 0.70)$



$\overline{V}_*^{-1}$

Modes on turbulent plasma:

$$\omega = k_y V_*^{eff}$$

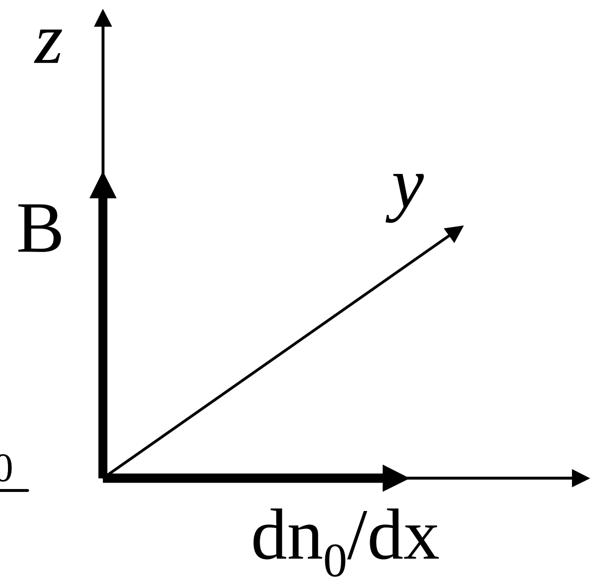
$$\gamma = \frac{\sqrt{\pi}}{|k_z| v_{Te}} \frac{k_y^2 (V_* - V_*^{eff})(V_*^{eff} - \frac{n_{tr}}{n_f} V_*)}{2 - \Gamma_0 \Im} - k_i^2 D_i \frac{2 - \Gamma_0 \Im n_{tr}}{2 - \Gamma_0 \Im} + k_i k_j R_{ij} V_*^{eff}$$

$$V_*^{eff} = V_* \frac{\Gamma_0 \Im (n_f - n_{tr}) + 2 n_{tr}}{2 - \Gamma_0 \Im}$$

$$\Im = \exp\left(-\frac{1}{2} k_i^2 S_i^2 (\overline{V_*}) \right)$$

$$R_{ij}(\tau, t) = \int_\tau^t d\theta' \int_{-\infty}^{\tau-\theta'} d\theta M_{ij}(|\theta|), \quad M_{ij}(|\theta|) = \left\langle v_i(0,0) \frac{\partial}{\partial y} v_j(\vec{x}(\theta), \theta) \right\rangle$$

Smooth density (quiescent plasma):



- The instability is produced by the combined effect of resonant electrons and finite Larmor radius (FLR) of the ions;

$$\omega = k_y V_*^{eff}$$

$$\gamma = \frac{\sqrt{\pi}}{|k_z| v_{Te}} \frac{k_y^2 (V_* - V_*^{eff}) V_*^{eff}}{2 - \Gamma_0}$$

$$V_*^{eff} = V_* \frac{\Gamma_0}{2 - \Gamma_0}$$

$$V_* = \frac{T}{e n_0 B} \frac{dn_0}{dx}$$

$$\Gamma_0 = \Gamma_0 \left(\frac{k_\perp^2 \rho_{Li}^2}{2} \right)$$

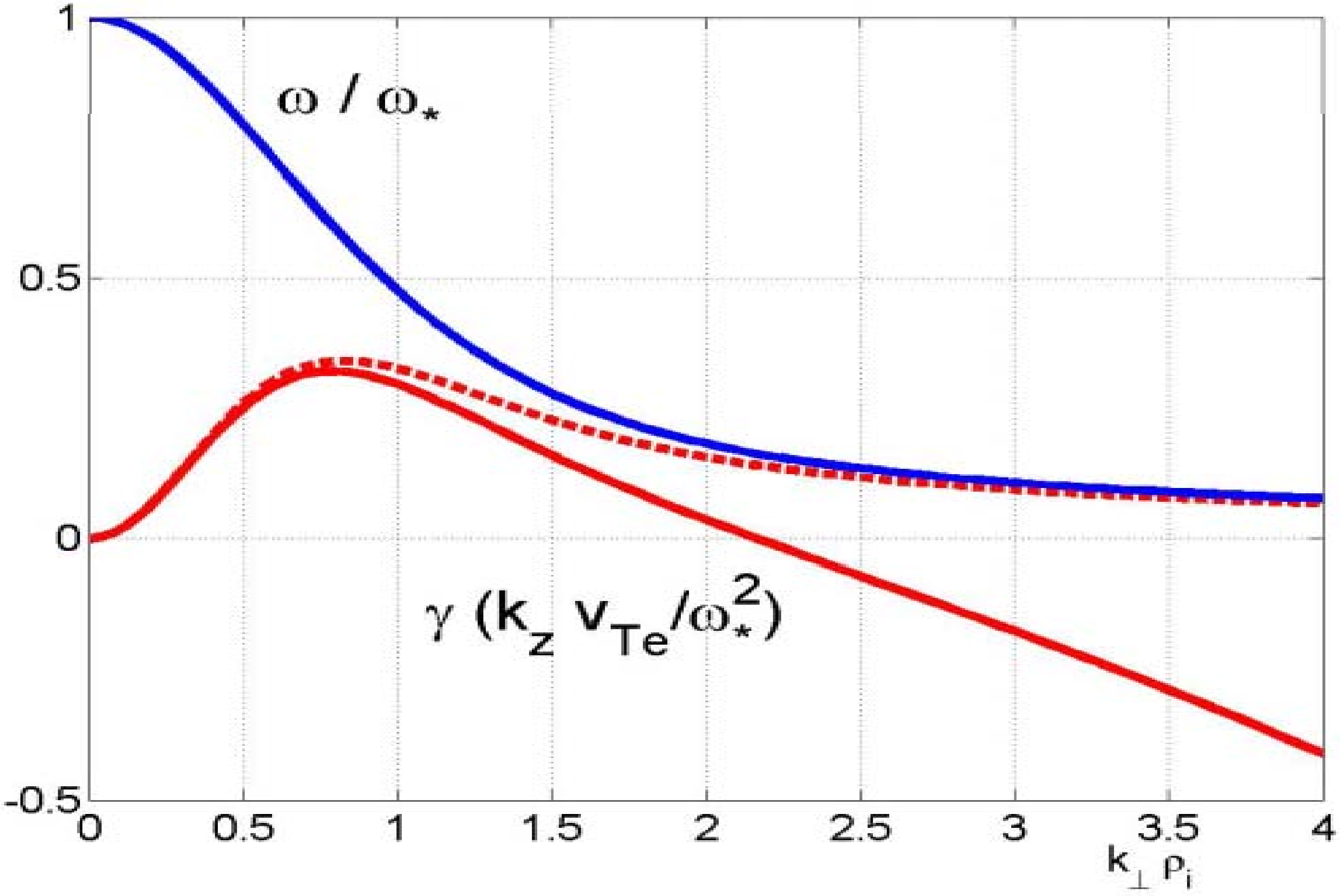
Turbulent plasma with small amplitude (quasilinear turbulence $V < V_*$):

$$\gamma = \frac{\sqrt{\pi}}{|k_z| v_{Te}} \frac{k_y^2 (V_* - V_*^{eff}) V_*^{eff}}{2 - \Gamma_0} - \frac{2 k_i^2 D_i}{2 - \Gamma_0}$$

Damping due to ion diffusion (resonance broadening of Dupree)

$V < V_*$ **Damping of the test modes with large k**

(resonance broadening - Dupree)



Turbulent plasma with higher amplitude $V \geq V_*$

Trapping appears but $n_{tr} << n_f$

The first nonlinear effect comes from trajectory vortical structures, which modify the effective diamagnetic velocity with the factor:

$$\Im = \exp\left(-\frac{1}{2} k_i^{\,2} S_i^{\,2} (\bar{V}_*) \right)$$

Effect of ion trajectory structures

$$\omega = k_y V_*^{eff}$$

$$\gamma = \frac{\sqrt{\pi}}{|k_z| v_{Te}} \frac{k_y^{\,2}(V_* - V_*^{eff}) V_*^{eff}}{2 - \Gamma_0 \Im} - k_i^{\,2} D_i \frac{2}{2 - \Gamma_0 \Im}$$

$$V_*^{eff} = V_* \frac{\Gamma_0 \Im}{2 - \Gamma_0 \Im}$$

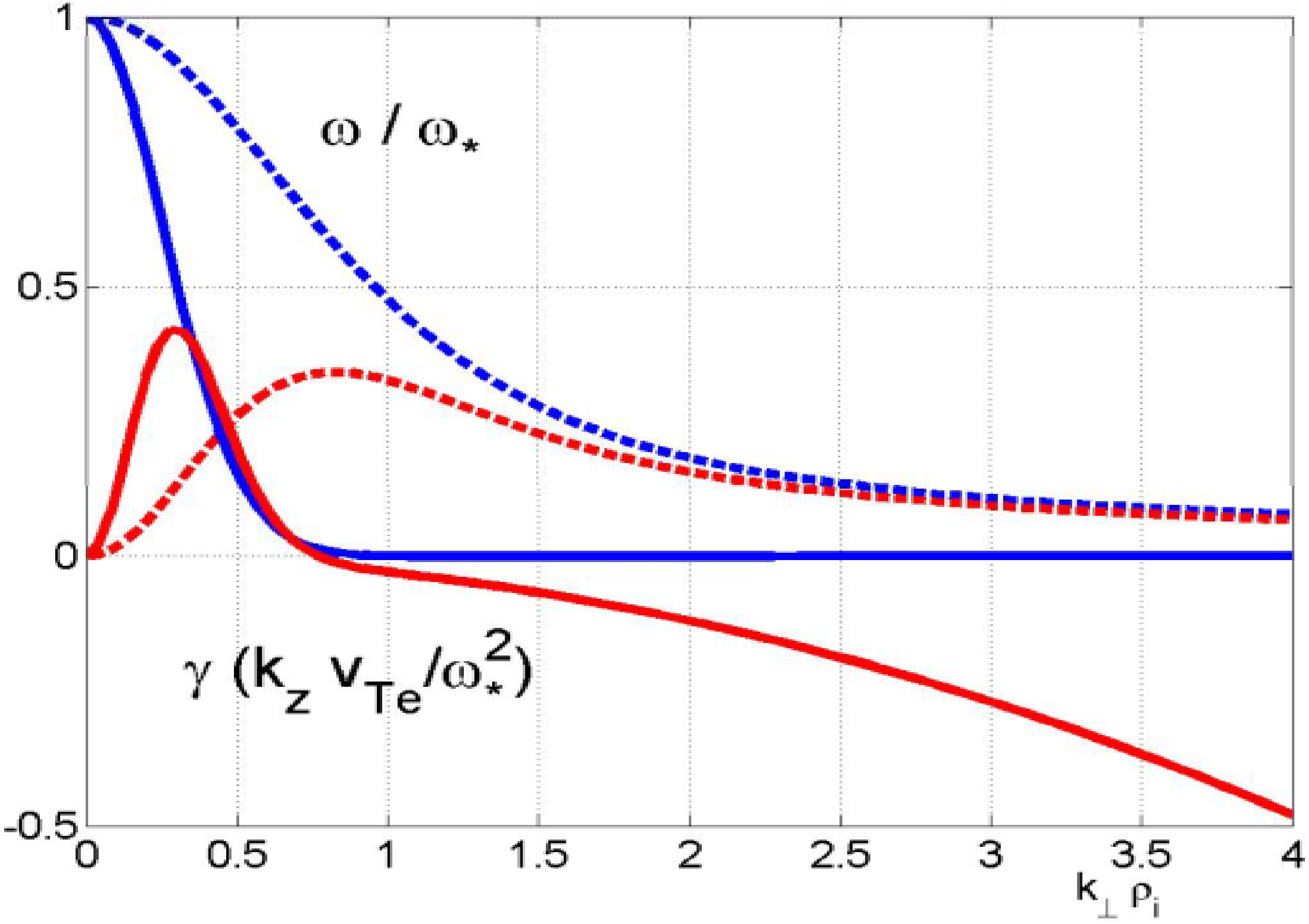
Decrease of the effective diamagnetic velocity

$V \geq V_*$

$n_{tr} << n_f$

Displacement of the unstable range toward small k due to trajectory structures while the maximum growth rate is not changed $\longrightarrow$ Both the amplitude of the turbulence and correlation length increase

Inverse cascade appears as the shift of the unstable wave numbers



Turbulent plasma very high amplitude $V >> V_*,\, n_{tr} \approx n_f$

- 1) The ion flows induced by the moving potential become important at this stage
  $\longrightarrow$ ***increase*** of the effective diamagnetic velocity

$$n_{tr} \to n_f \qquad V_*^{eff} = V_* \frac{\cancel{\Gamma_0 \Im (n_f - n_{tr})} + 2n_{tr}}{2 - \Gamma_0 \Im} \to V_*$$

$$\gamma_+ = \frac{\sqrt{\pi}}{|k_z| v_{Te}} \frac{k_y{}^2 (V_* - V_*^{eff})(V_*^{eff} - \frac{n_{tr}}{n_f} V_*)}{2 - \Gamma_0 \Im} \to 0$$

$$\gamma_{\max} = \frac{\sqrt{\pi}}{4|k_z| v_{Te}} k_y{}^2 V_*{}^2 (1 - \frac{n_{tr}}{n_f})^2 \to 0 \quad for \quad n_{tr} = 1/2$$

The evolution of the amplitude becomes slower and eventually the growth rates vanishes; the wave numbers and the frequencies are small $\longrightarrow$ damping by the parallel motion of the ions.

- 1) The fluctuation of the diamagnetic velocity due to background turbulence determine a direct contribution to the growth rate

$$\gamma = \frac{\sqrt{\pi}}{|k_z| v_{Te}} \frac{k_y^{\,2}(V_* - V_*^{eff})(V_*^{eff} - \frac{n_{tr}}{n_f} V_*)}{2 - \Gamma_0 \Im} - k_i^{\,2} D_i \frac{2 - \Gamma_0 \Im n_{tr}}{2 - \Gamma_0 \Im} + k_i k_j R_{ij} V_*^{eff}$$

$$R_{ij}(\tau, t) = \int_\tau^t d\theta' \int_{-\infty}^{\tau-\theta'} d\theta\, M_{ij}(|\theta|), \quad M_{ij}(|\theta|) = \left\langle v_i(0,0) \frac{\partial}{\partial y} v_j(\vec{x}(\theta), \theta) \right\rangle$$

- This term is zero for homogeneous and izotropic turbulence and strongly depends on the parameters of the anizotropy.
- The i=j=1 component corresponds to zonal flows. It appears for trapped particles due to the anizotropy induced by particle flows with the moving potential.

$$\omega = k_y V_*^{eff}$$

(1) Dupree turbulent damping (V>0)

$$\gamma = \frac{\sqrt{\pi}}{|k_z| v_{Te}} \frac{k_y^{\,2}(V_* - V_*^{eff})\left(V_*^{eff} - \boxed{\frac{n_{tr}}{n_f}} V_*\right)}{2 - \Gamma_0 \Im} - k_i^{\,2} D_i \frac{2 - \Gamma_0 \Im n_{tr}}{2 - \Gamma_0 \Im} + k_i k_j R_{ij} V_*^{eff}$$

(3) Fluctuations of V_*

$$V_*^{eff} = V_* \frac{\Gamma_0 \Im \boxed{(n_f - n_{tr})} + \boxed{2n_{tr}}}{2 - \Gamma_0 \Im}$$

(3) Ion flows $V >> V_*,\ n_{tr} \approx n_f$

$$\Im = \exp\left(-\frac{1}{2} k_i^{\,2} S_i^{\,2}(V)\right)$$

(2) Trajectory structures

$V > V_*,\ n_{tr} << n_f;\quad \Gamma_0 \to \Gamma_0 \Im$

$$R_{ij}(\tau, t) = \int_\tau^t d\theta' \int_{-\infty}^{\tau-\theta'} d\theta\, M_{ij}(|\theta|), \quad M_{ij}(|\theta|) = \left\langle v_i(0,0) \frac{\partial}{\partial y} v_j(\vec{x}(\theta), \theta) \right\rangle$$

Conclusion

- ***Trapping determines a strong and complex influence on test modes in turbulent plasmas producing large scale potential cells and zonal flows.***