

# Modeling Solar Wind Turbulence: The Kolmogorov Way

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Study of solar wind turbulence

Direct observations

Collisionless plasma, wave-wave interactions

Propagation of cosmic rays

Scattering of particles on fluctuations

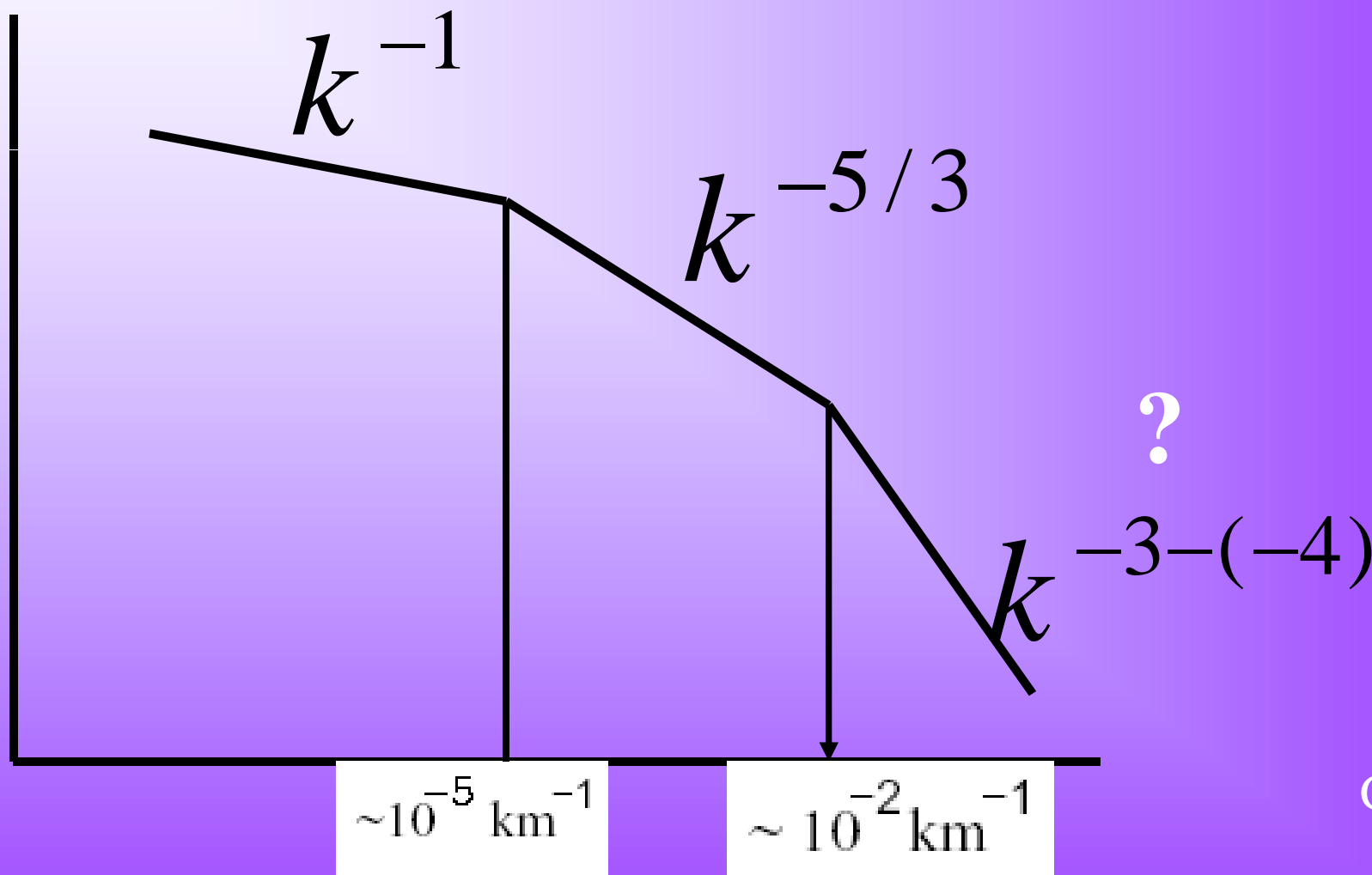
Solar –terrestrial relation

Coupling of SW with earth's magnetosphere,  
anomalous viscosity

Several exponents on the short spatial scales

Lessons for other astrophysical systems

# SCHEMATIC MAGNETIC SPECTRA IN THE SOLAR WIND



DISSIPATIVE  
DISPERSIVE  
OR  
INERTIAL ?

Goldstein, Roberts  
Mathaeus

Annu. Rev of  
Astronomy &  
Astrophysics,  
1995

Reduced Spectra by averaging over the two directions perpendicular to the solar wind velocity. Spectra are a function of the wavenumber along VSW.

# Density Fluctuations

Are they generated by the same mechanisms  
as the velocity and the magnetic field fluctuations ?

**Alfven waves do not generate density fluctuations!**

**Magnetosonic Waves do!**

***OR***

Are they created by some other mechanism  
**AND** convected by the  $v, b$  flow as a passive scalar ?

**Spectrum would distinguish these cases!**

**One Proposal is to  
assume a  $k^{-1}$  spectrum of Alfvén waves**

**and**

**its cascade produces a  $k^{-5/3}$  spectrum**

**Steepened part**

**due to damping of kinetic Alfvén waves**

**ISSUES**

**How is the  $k^{-1}$  spectrum produced?**

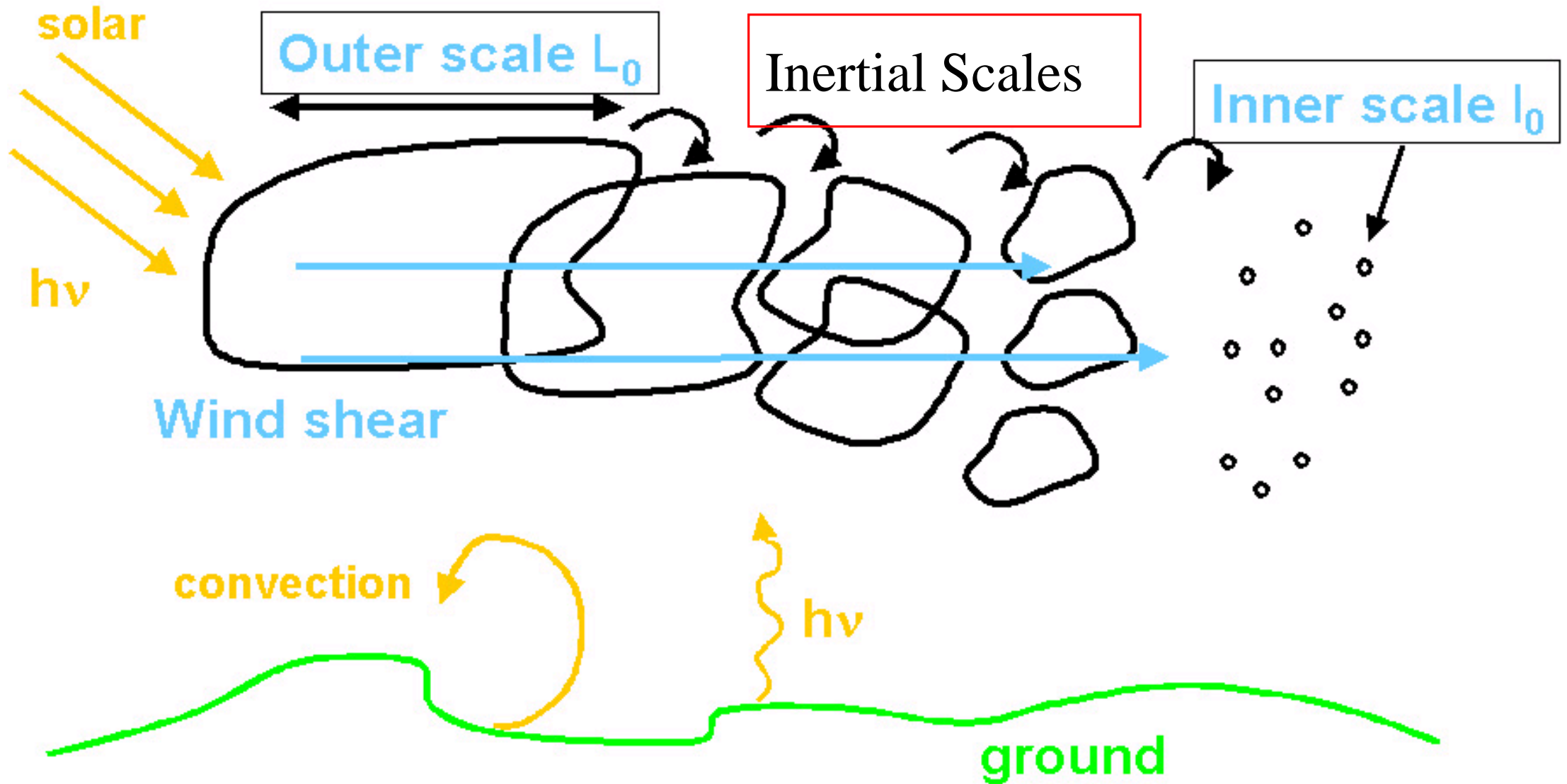
**Damping mechanisms generate steeper than  
the observed spectra!**

**OUR Proposal is through  
the Kolmogorovic Dimensional Arguments  
Identify the Invariants  
Their Kolmogorovic cascade  
produces  
spectra with different exponents**

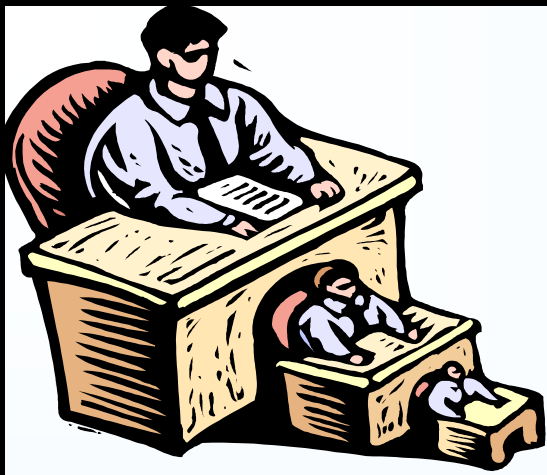
**As e.g. in 2D Hydrodynamic Turbulence  
with Energy and Enstrophy as the two invariants  
resulting in  $k^{-5/3}$  and  $k^{-3}$  spectra**

# Kolmogorov Hypothesis (Andrey Nikolaevich Kolmogorov)

Cascade from large eddies to small eddies



# CASCADES



In order to derive the spectral energy distributions

we resort to the Kolmogorov

**HYPOTHESES**

the spectral cascades proceed at a constant rate governed

**By the Eddy Turn Over Time**

$$(kV_k)^{-1}$$

# Illustration of the Kolmogorov Way or the Dimensional Arguments

In 3-D homogeneous and isotropic turbulence

## The Energy Spectrum in INERTIAL RANGE

$$(kV_k)(V_k^2) = \varepsilon_v$$

$$E(k) = \frac{(V_k)^2}{k}$$

$$E(k) = \varepsilon^{2/3} k^{-5/3}$$

# Incompressible Alfvénic Turbulence in the Inertial Range

## THE TWO MHD INVARIANTS

$$\text{Total Energy} \quad E = \frac{1}{2} \int (V^2 + B^2) d^3x$$

$$\text{Magnetic Helicity} \quad H_M = \frac{1}{2} \int A \cdot B d^3x$$

Stribling et al.  
1994, 99,2567

Cascade of  
Magnetic Helicity

$$(kV_k) \left( \frac{B_k^2}{k} \right) = \varepsilon_H, \quad V_k^2 = B_k^2$$

$$E(k) = (\varepsilon_H)^{2/3} k^{-1}$$

# Incompressible MHD Turbulence -----> Alfvenic Fluctuations

$$\delta v = \pm \delta b, \quad \delta n = 0, \quad k = \pm k_z$$

Cascading Properties of the two invariants

The Total Energy and The Magnetic Helicity

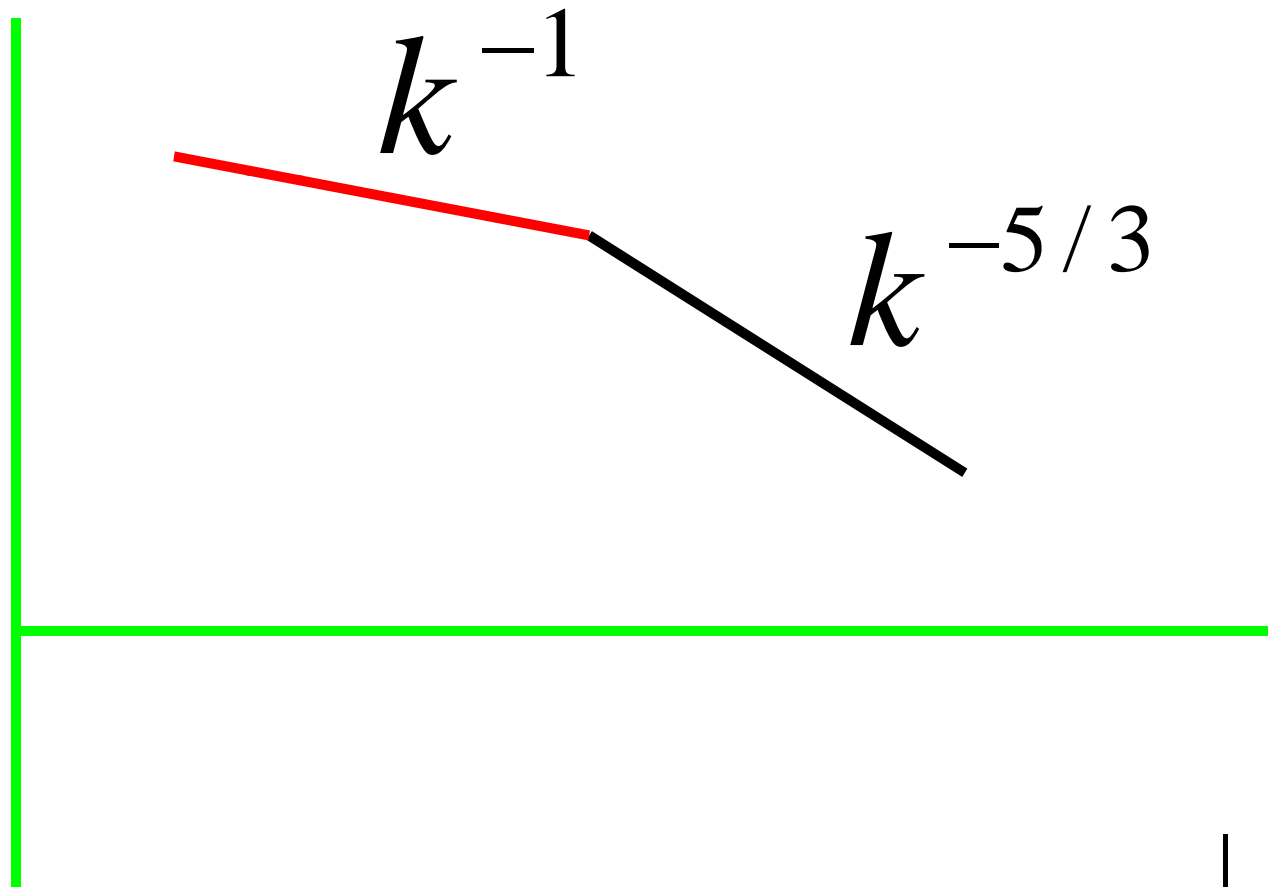
Produce the identical v,b spectra in the inertial range

Density fluctuation are just convected by the flow like the corks in a fluid !

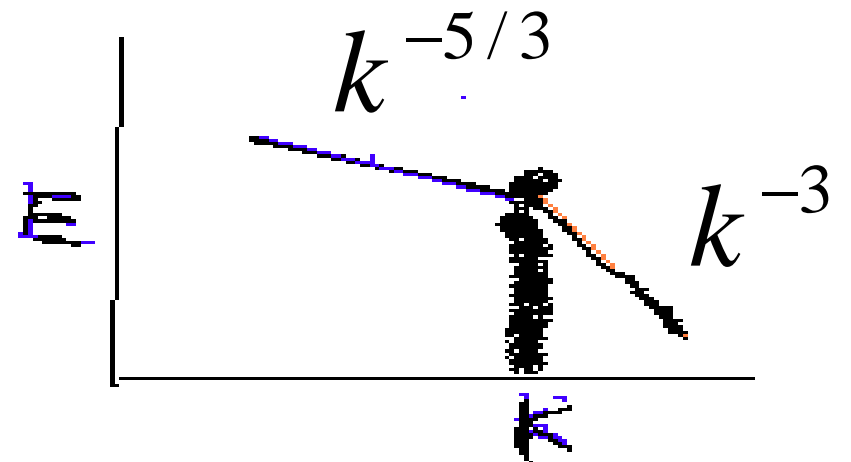
# Incompressible Alfvenic Turbulence in the Inertial Range

$V=B$ , Identical spectra, Density fluctuations are just carried by  $V, B$

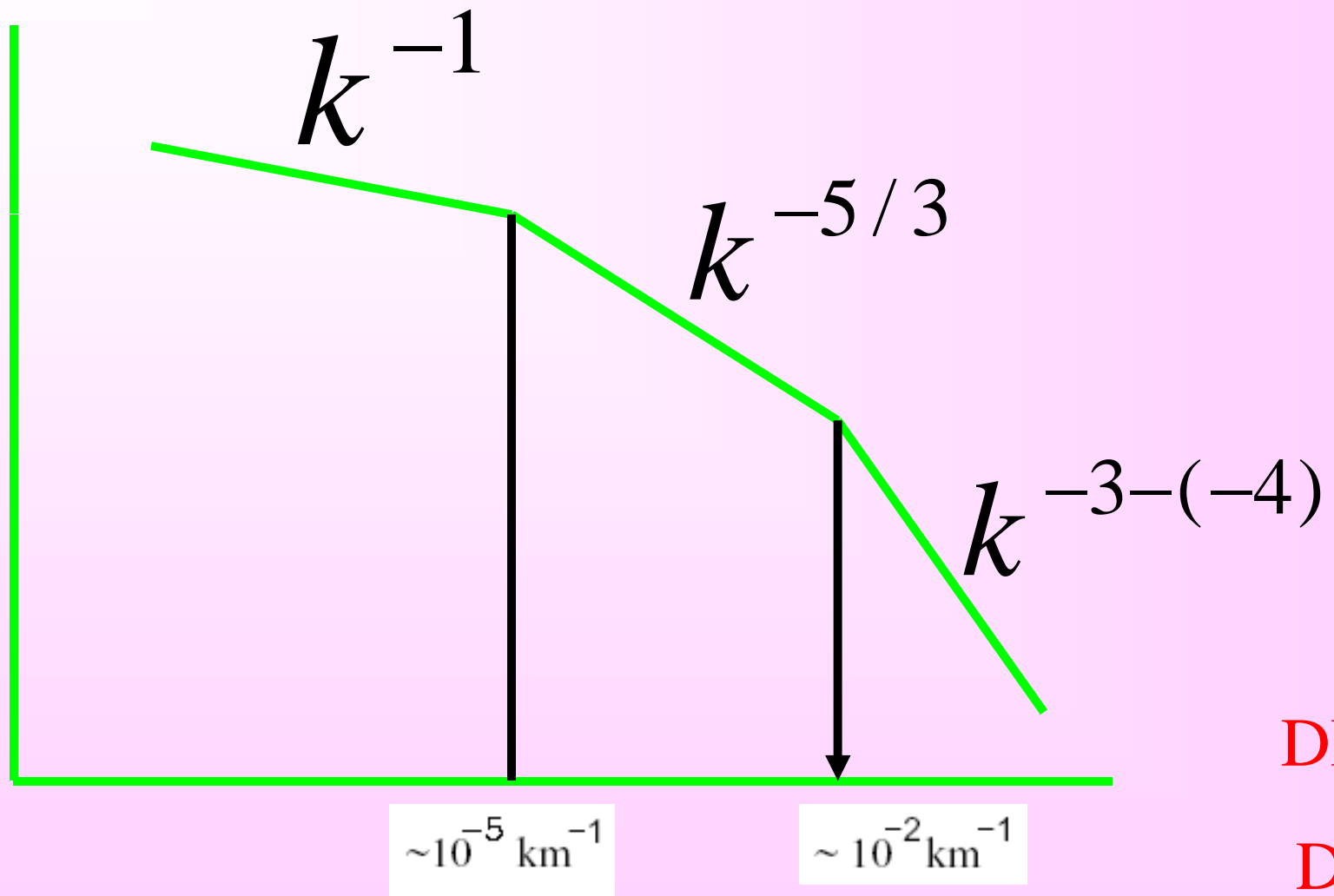
$E_m(k)$   
 $E_v(k)$   
 $E_n(k)$



AS IN 2-D



# SCHEMATIC MAGNETIC SPECTRA IN THE SOLAR WIND



?

DISSIPATIVE  
DISPERSIVE  
OR  
INERTIAL ?

# *Inclusion of the Hall-Effect in the generally accepted Alfvenic Turbulence*

Why Hall ?

Steepening observed near the Ion Inertial scale  $\lambda_i = \frac{c}{\omega_{pi}}$

Towards short scales

Dispersion caused by the Hall effect introduces k dependence in the V B relation around these scales

Different Spectra for V and B

What convects the density fluctuations? V or B ?

Departures from Alfvenicity also observed at short scales

# THE HALL-MHD

The two –Fluid Model

$$m_i n_i \left[ \frac{\partial V_i}{\partial t} + (V_i \cdot \nabla) V_i \right] = \frac{J \times B}{c}$$

$$\frac{\partial B}{\partial t} = \nabla \times \left[ \left( V_i - \frac{J}{en} \right) \times B \right]$$

*The Hall term decouples electron and ion motion on ion inertial length scales and ion cyclotron times*

# Waves in Hall-MHD

$$\vec{v} - \varepsilon \nabla \times \vec{b} = -\frac{\omega}{k} \vec{b}$$

$$\nabla \times \vec{b} = -\frac{\omega}{k} \nabla \times \vec{v} \quad b_k = \alpha(k) v_k$$

*and*  $\alpha(k) \equiv \left(-\frac{\omega}{k}\right) = \left[-k/2 \pm (k^2/4 + 1)^{1/2}\right]$

NOTICE DEPARTURES FROM THE ALFVEN STATE

FOR WHICH

$$v = \pm b$$

# THE THREE INVARIANTS OF THE HALL-MHD

$$\text{Total Energy } E = \frac{1}{2} \int (V^2 + B^2) d^3x = \frac{1}{2} \sum_{\mathbf{k}} |V_{\mathbf{k}}|^2 + |B_{\mathbf{k}}|^2$$

$$\text{Magnetic Helicity } H_M = \frac{1}{2} \int \mathbf{A} \cdot \mathbf{B} d^3x = \frac{1}{2} \sum_{\mathbf{k}} \frac{i}{k^2} (\mathbf{k} \times \mathbf{B}_{\mathbf{k}}) \cdot \mathbf{B}_{-\mathbf{k}}$$

$$\text{Generalized Helicity } H_G = \frac{1}{2} \int (\mathbf{A} + \mathbf{V}) \cdot (\mathbf{B} + \nabla \times \mathbf{V}) d^3x$$

$$= \frac{1}{2} \sum_{\mathbf{k}} \left[ \frac{i\mathbf{k} \times \mathbf{B}_{\mathbf{k}}}{k^2} + \mathbf{V}_{\mathbf{k}} \right] \cdot [\mathbf{B}_{-\mathbf{k}} - i\mathbf{k} \times \mathbf{V}_{-\mathbf{k}}]$$

# *SPECTRA IN THE HALL STATE*

*From The Hall relation*

$$V_k = [\alpha(k)]^{-1} B_k$$

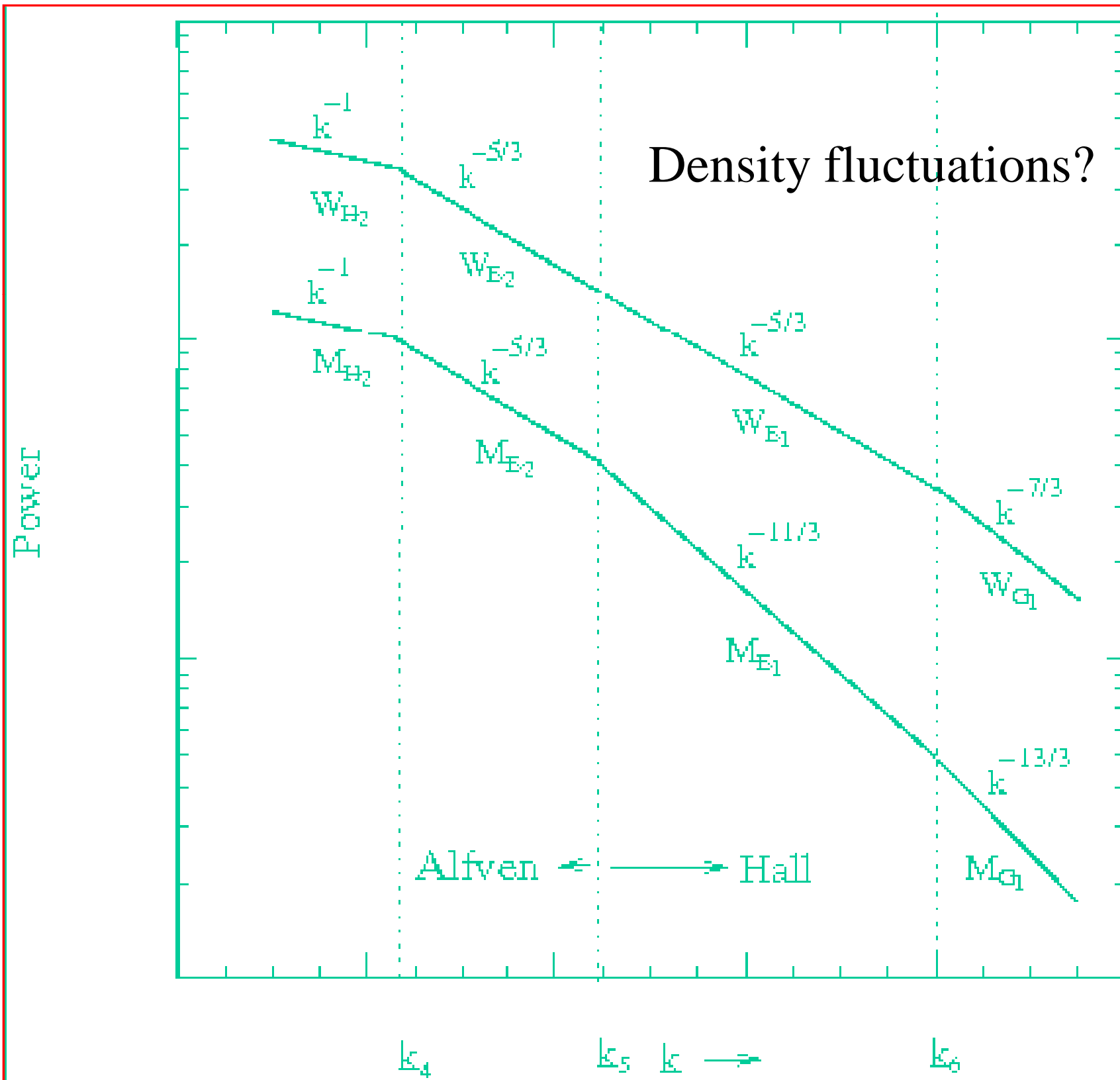
*For  $k \gg 1$ ,*

*$\alpha(k) \approx k^{-1}$  the shear Alfven branch*

*and*

*$\alpha(k) \approx k$  the whistler branch*

# Modeled solar wind spectra including Hall effect (for shear Alfvén branch)



$$b_k = k^{-1} v_k$$

Krishan, V.  
Mahajan, S. M.

Journal of  
Geophysical  
Research,  
Volume 109,  
Issue A11,  
A11105, 2004

What convects the density fluctuations? V or B ?

*B is frozen to electrons*

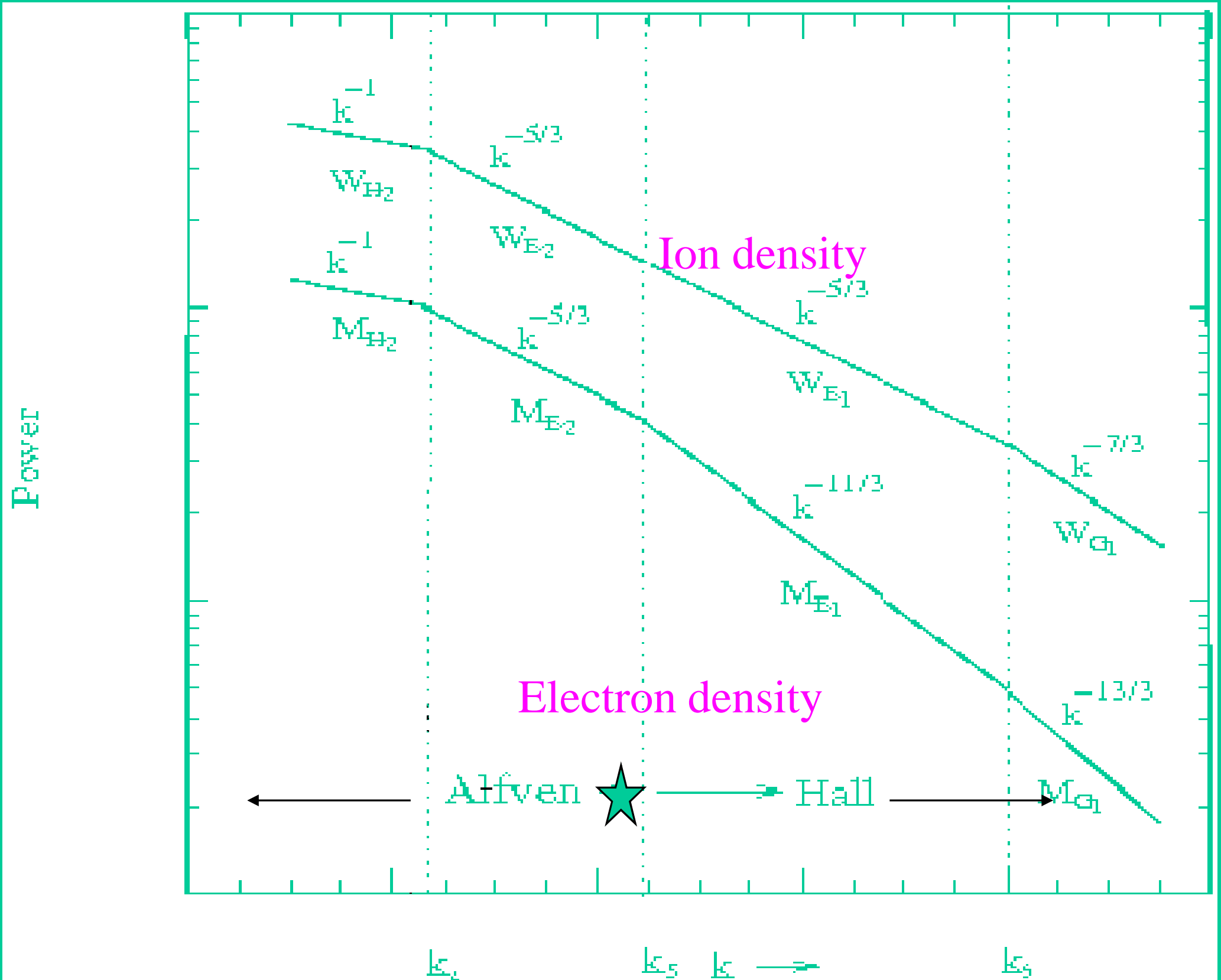
Therefore the electron density fluctuations will follow the magnetic fluctuation spectrum

*B is not frozen to ions*

Therefore the ion density fluctuations will follow the Velocity fluctuation spectrum

The two fluid effects in their full glory!

# Modeled solar wind spectra including Hall effect



**Tracking of the magnetic spectrum by the density spectrum is essentially a statistical form of pressure balance between the thermal and the magnetic pressure**

**Thus in the Hall regime if the electron density fluctuations are frozen to the magnetic field fluctuations**      **We must have**

$$\frac{b^2}{8\pi} \approx n_e k_B T_e$$

**And if the kinetic energy spectrum is tracked by the ion density fluctuations, there must be a balance between the dynamical pressure and the thermal pressure of the ions**

$$\frac{\rho_i v_i^2}{2} \approx n_i k_B T_i$$

**For shear Alfvén waves in the Hall regime**       $\frac{b_k^2}{8\pi} < \frac{1}{2} \rho_i v_k^2, \quad n_e = n_i$

**Therefore**       $T_e < T_i$

# Conclusion

Steepened spectra due to The Hall Effect

Electron density and Ion density Spectra differ

Dissipation range at even higher  $k$

Could be due to various wave-particle damping mechanisms.

Further  
investigation

Compressibility Effects, anisotropy, radial variation etc.

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# ORDERING OF VARIOUS SPATIAL SCALES

$$\beta < \frac{m_e}{m_i}$$

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Ideal  
MHD

$\lambda_{Le}$        $\lambda_{Li}$        $\lambda_e$        $\lambda_i$        $\infty$

$$\frac{m_e}{m_i} < \beta < 1$$

---

$\lambda_{Le}$        $\lambda_e$        $\lambda_{Li}$        $\lambda_i$        $\infty$

$$\beta > 1$$

---

$\lambda_e$        $\lambda_i$        $\lambda_{Le}$        $\lambda_{Li}$        $\infty$

Magnetic Helicity in the presence of external uniform field

$$H'_M = H_M + 2B_0 \cdot \langle v \times b \rangle$$

Three possibilities

1. The new term averages to zero over some timescale, it will not affect results based on inverse cascade of helicity. Scale separation, absence of alpha effect would also preserve the invariance of  $H_M$
2. The new term may become statistically equipartitioned with  $H_M$  with concomitant effects on spectral transfer
3.  $H_M$  might decay in favour of new term