Statistical Theory of Plasma Turbulence

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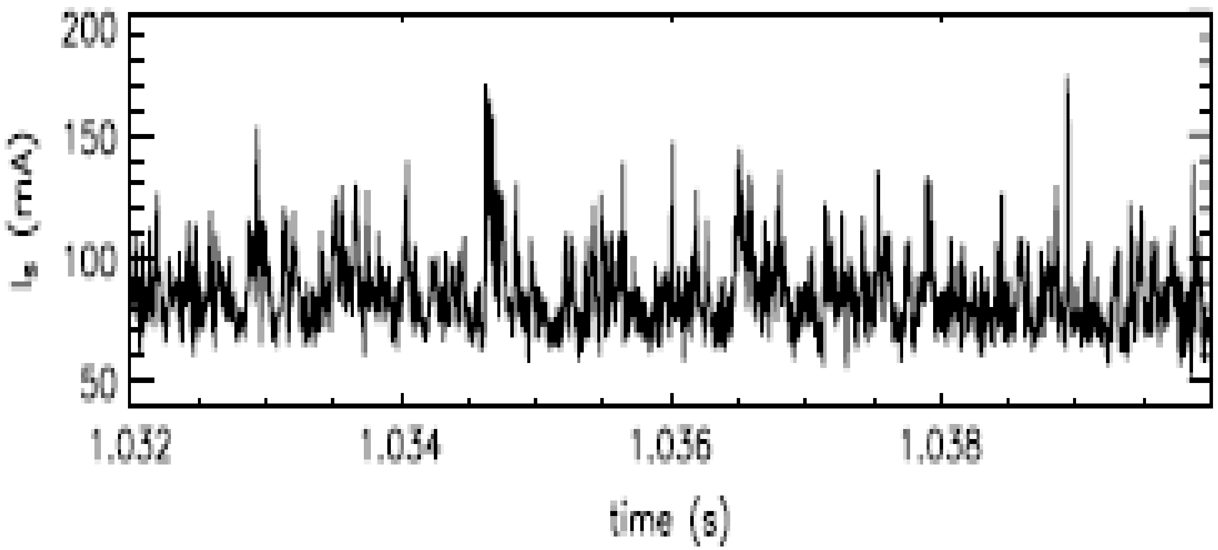
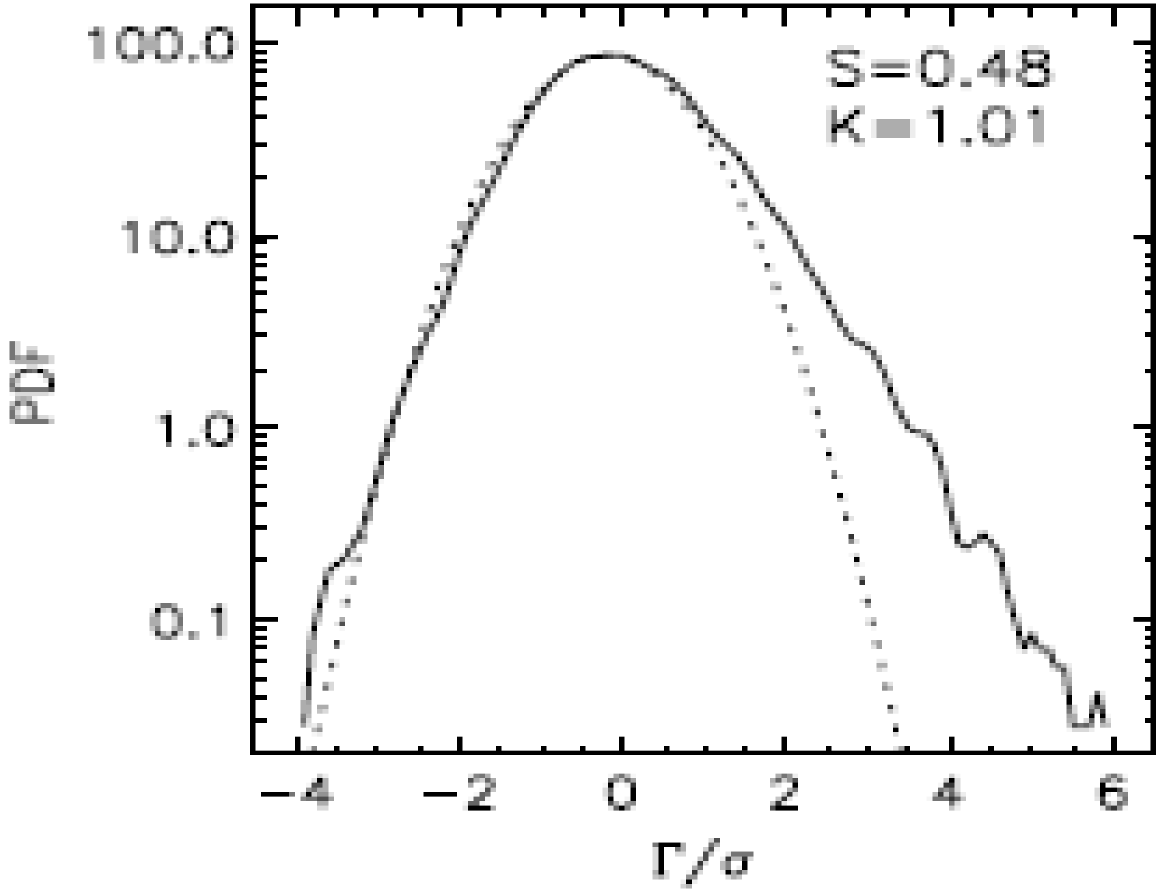
Future reactors

- Minimization of anomalous transport
- Controlling events of large amplitude





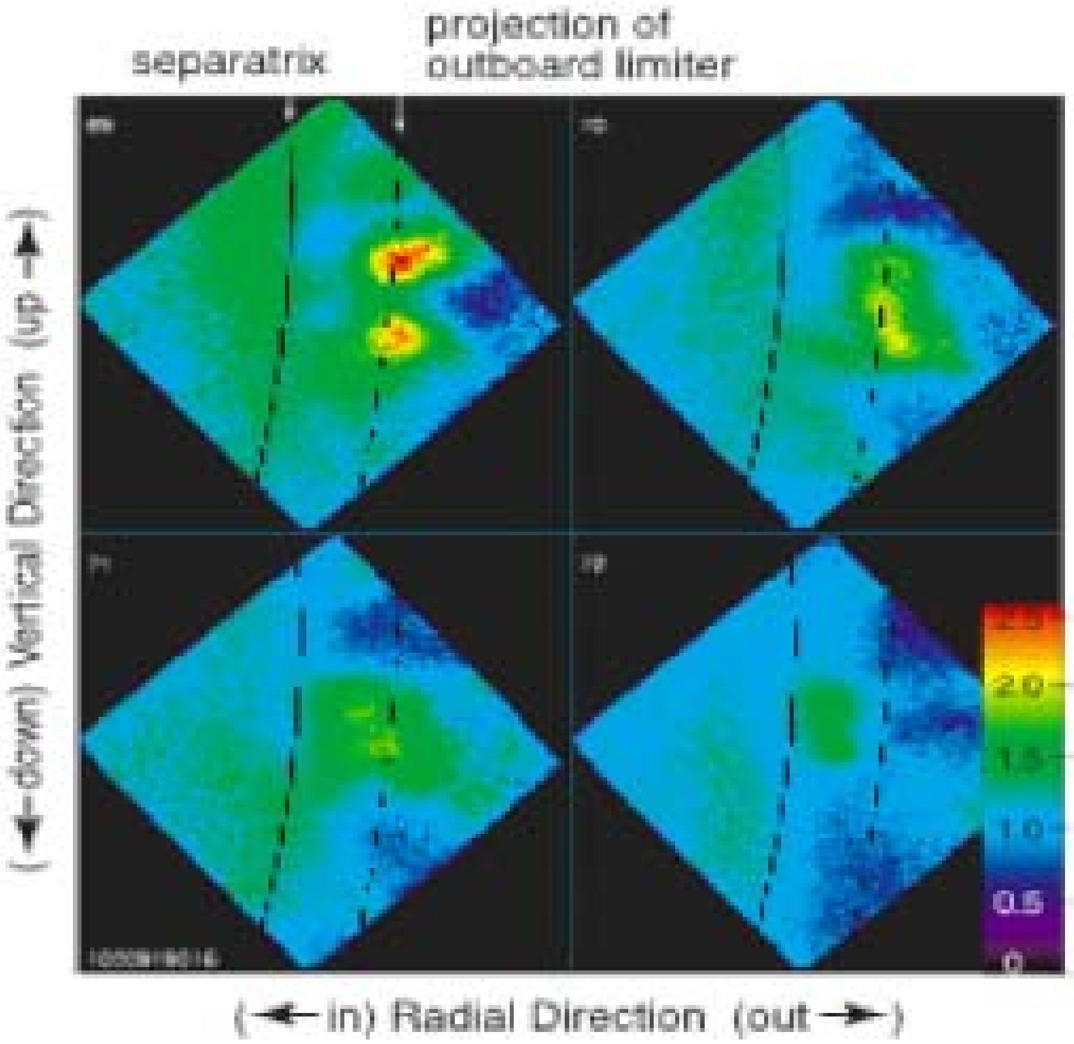
Edge turbulence in TEXTOR
[Xu et al, PPCF 06]

Observation

A. Short-lived coherent structures (e.g. blobs, vortices)

B. Intermittency in PDF tails: exponential [Zweben et al, 06]

Goal: universal theory of B based on A



Edge turbulence in Alcator C-Mod (Zweben et al, PPCF '07)

Outline

I. General structure-based statistical theory

II. Structure formation

III. Self-organization

IV. Conclusion

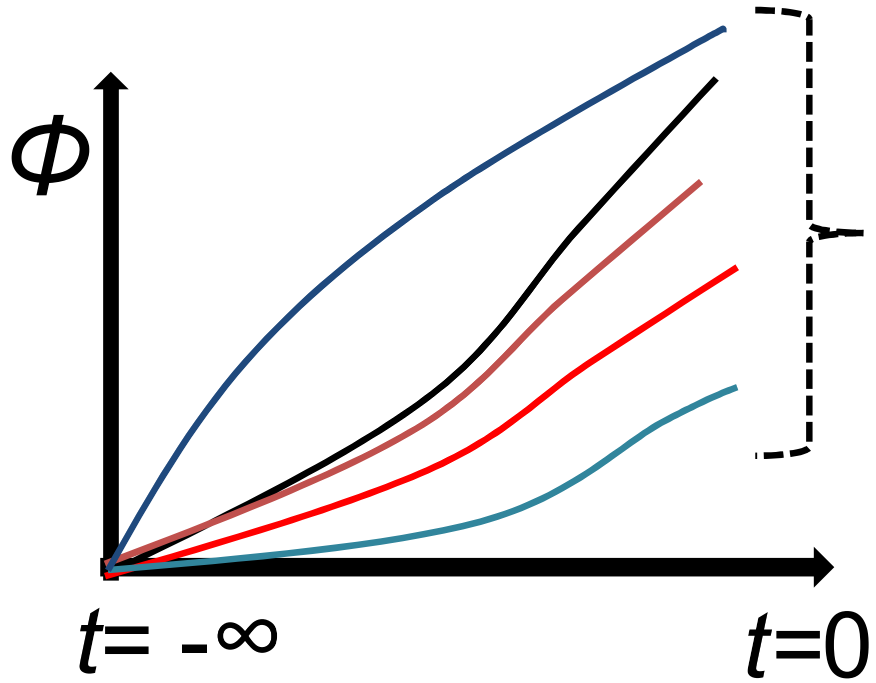
I. Structure-based theory

$$\partial_t \phi + \underline{N(\phi)} = \underline{f}$$

Linear/nonlinear int. forcing

Forcing: uncertainty in *Φ* and *M(Φ)*

- Φ =0 at t= -∞
- Φ=(-∞,∞) at t=0



PDF of *R=M(Φ)*

$$P(R)=\int d\lambda \exp(i\lambda R)\int D\phi D\bar{\phi}\exp(-S_\lambda)$$

$$S_\lambda=-i\int dxdt\bar{\phi}[\partial_t\phi+N(\phi)]+\frac{1}{2}\int dxdydt\bar{\phi}(x)\kappa(x-y)\bar{\phi}(y)$$

$$+i\lambda\int dxdtM(\phi)\delta(t)$$

where $\langle f(x,t)f(y,t')\rangle=\delta(t-t')\kappa(x-y)$

How to compute the path-integral?

I. Tradition: small perturbation around Φ=0

II. Non-perturbative theory:

i) Nontrivial vacuum $\Phi\neq0$ with largest probability

ii) Short-lived **coherent structure**

$$\phi(x,t) = F(t)\underline{\phi_0(x)}$$

"instantons"

↑ localized ↑ nonlinear solution

Stochastic forcing

→ coherent structures [non-trivial vacuum]

PDFs of $M(\Phi)$

$N(\Phi)$ involves the ***nth*** highest nonlinear interaction

$M(\Phi)=\langle \Phi\,\Phi\ldots\,\Phi\rangle$ is ***mth*** moment of Φ

$$P(R) \propto \exp[-cR^{s}]$$

$$s = \frac{n+1}{m}, c = \frac{\alpha}{\kappa_0}$$

[Kim & Anderson 08]

i) Exponential PDF tails with exponent ***s***

ii) Amplitude ***c*** depends on structure and forcing

$\alpha \propto$ overlap between ϕ_0 and $N(\phi_0)$

$\kappa_0 \propto$ overlap between ϕ_0 and forcing

I. Linear system: $s=2/m$ [m:moment]

$$m=1: \exp(-cR^2)$$ [Gaussian]

$$m=2: \exp(-cR)$$ [Stretched exponential]

II. Quadratic nonlinear system: $s=3/m$

$$m=1: \exp(-cR^3)$$

$$m=2: \exp(-cR^{3/2})$$

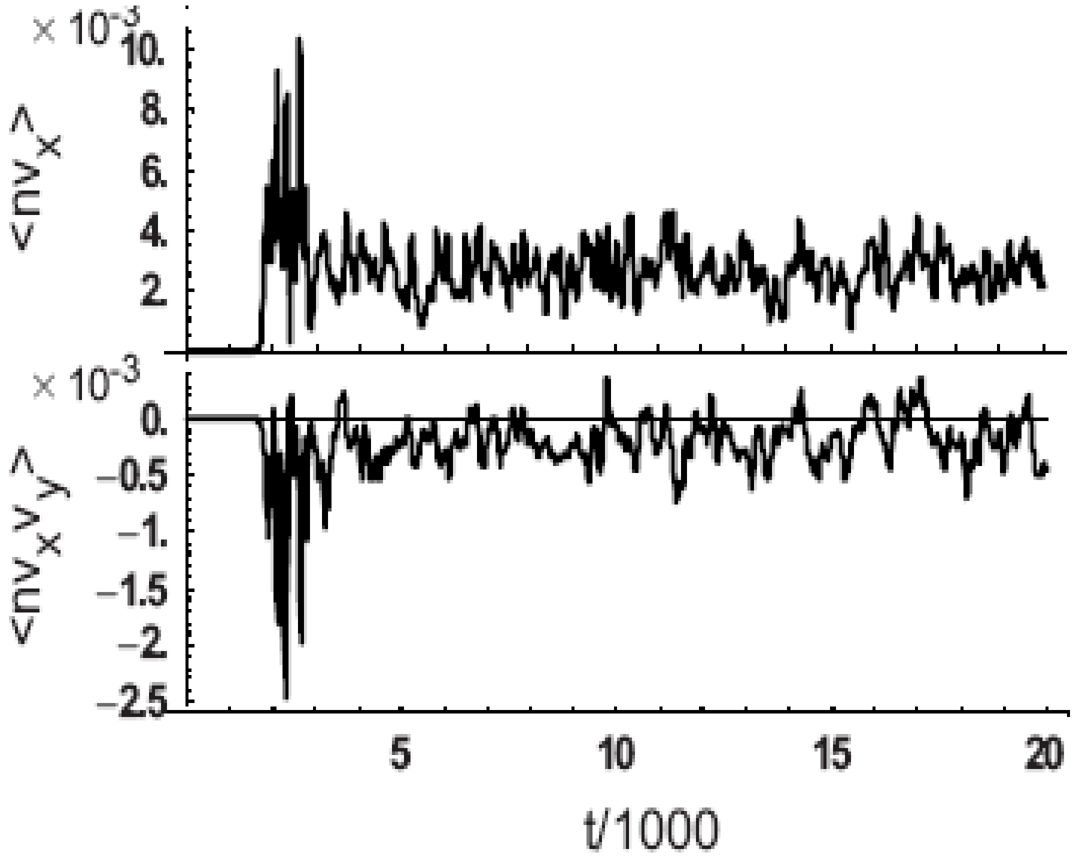
$$m=3: \exp(-cR)$$

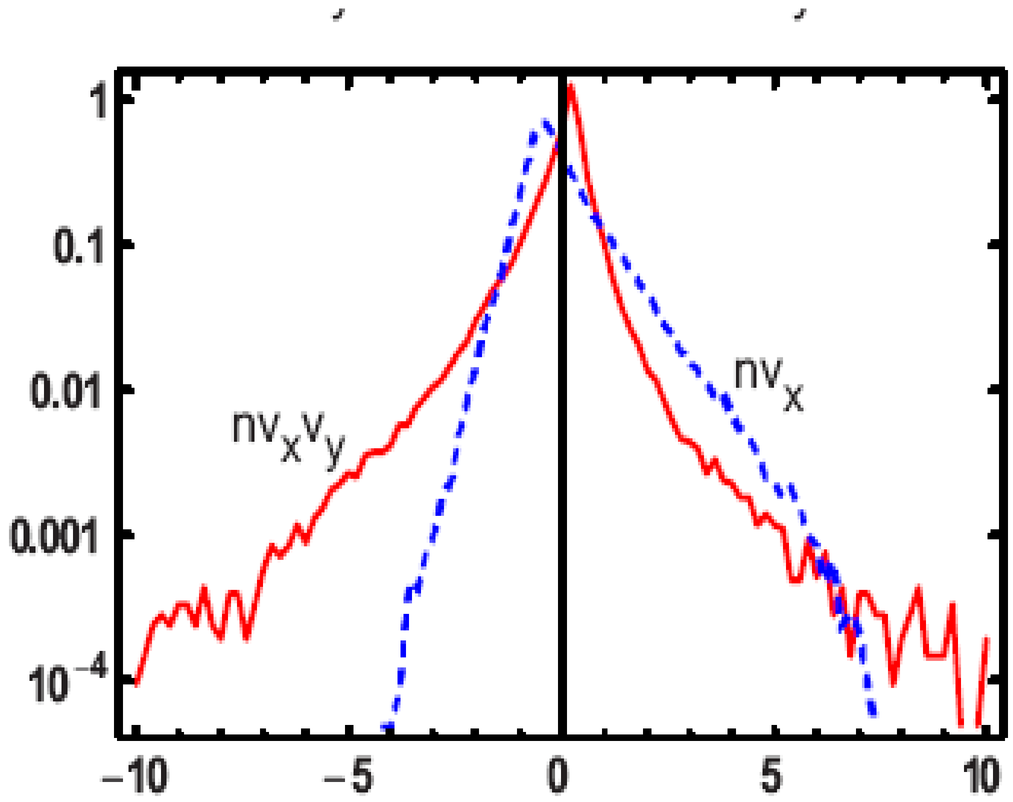
$$m=4: \exp(-cR^{3/4})$$

→PDF increases for higher moments

Myra et al, PoP 08

[edge turbulence in drift-interchange model]





$$v_x = -\partial_x \phi, v_y = \partial_y \phi$$

Exponential PDFs with ***s*<1**

II. Structure formation

- PDFs required for formation of structures
- PDFs for the L-H transition/ITB formation
- Zonal flows driven by Reynolds stress

$$\partial_t \phi_{ZF} = \langle \partial_x \phi \partial_y \phi \rangle \text{ + (damping)}$$

i) PDF of ϕ_{ZF}

ii) PDF of $\langle -\partial_x \phi \partial_y \phi \rangle$

A. Momentum transport

PDFs of local momentum flux R

•Hasagawa-Mima [Kim and Diamond PRL 02; PoP 02]

•Toroidal ITG turbulence [Kim et al, Nucl. Fusion, 03]

PDFs of averaged momentum flux R

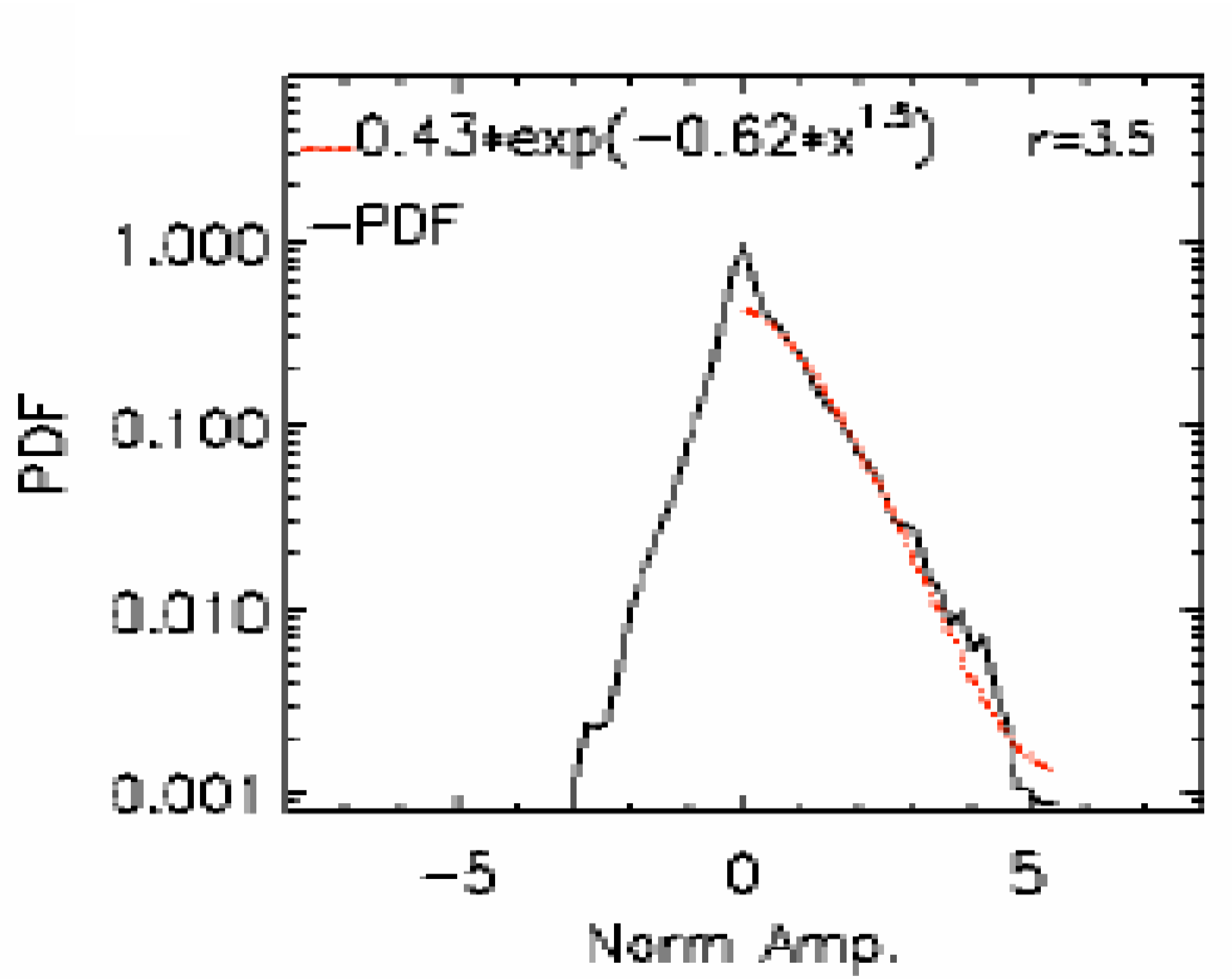
• ITG turbulence [Anderson and Kim, PoP 15, 052306 (2008)]

Coherent structures: modons

$$T_i \propto \phi$$

$$\Rightarrow \exp(-cR^{3/2})$$

$$[n = 2, m = 2]$$

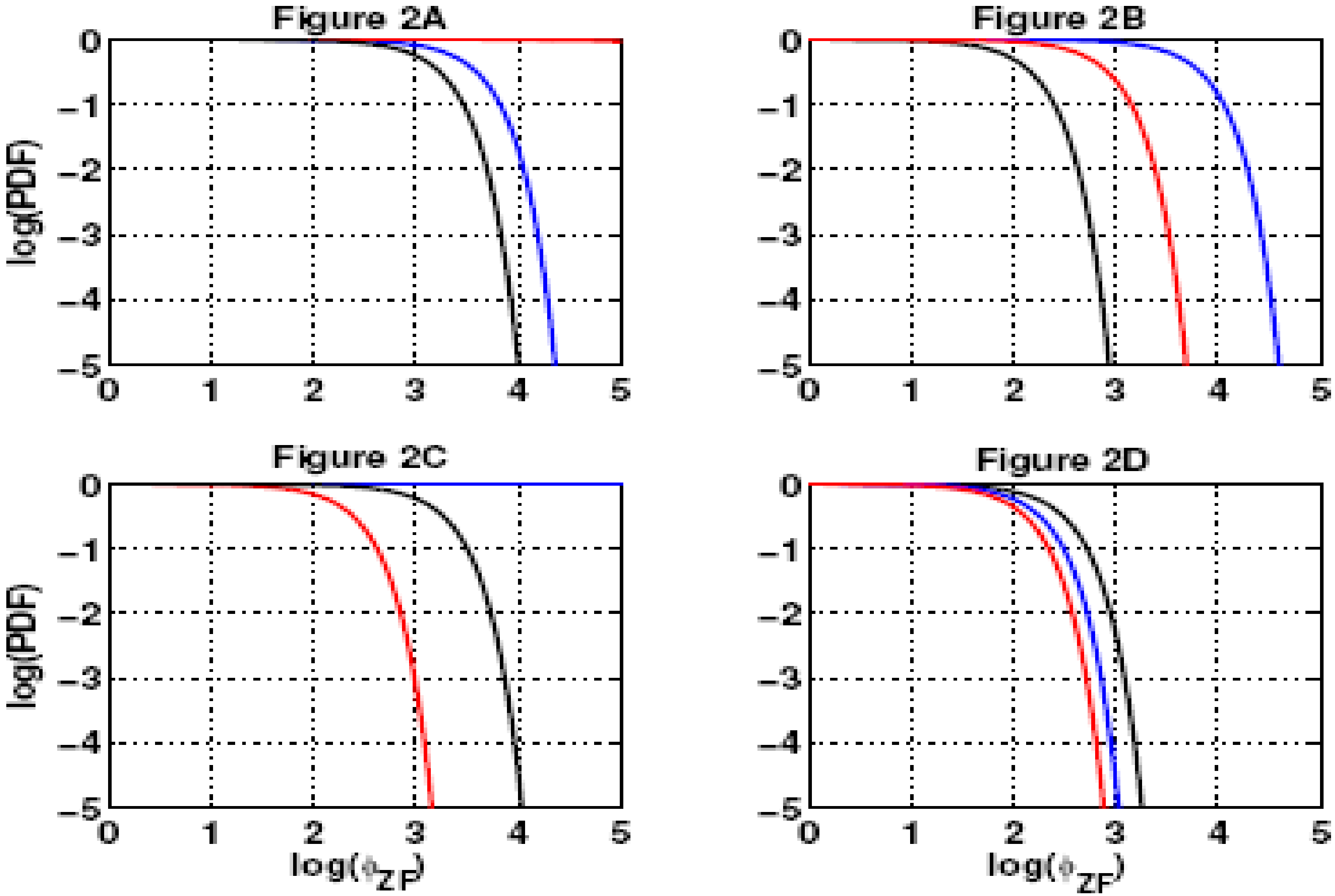


Yan et al, APS-DPP 08

B. Zonal flow formation

[Anderson and Kim, PoP, 15, 082312 (2008)]

$$\partial_t \phi_{ZF} = \langle \partial_x \phi \partial_y \phi \rangle \Rightarrow P(\phi_{ZF}) \propto \exp(-c\phi_{ZF}^3)$$



III. Self-organization

[Kim, Liu and Anderson, in preparation 08]

$$\partial_t u = \partial_x [D(\partial_x u)\partial_x u] + f$$

Model A

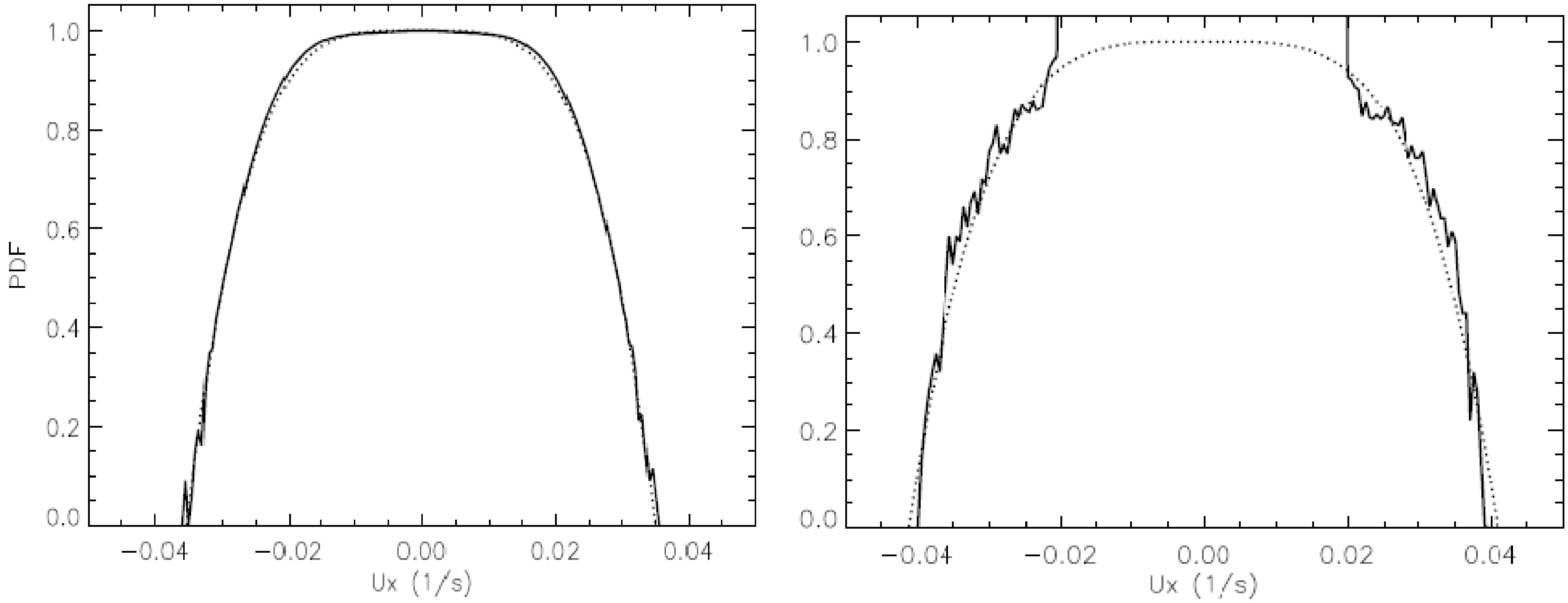
$$D(\partial_x u) \propto (\partial_x u)^2$$

Model B

$$D(\partial_x u) = V \,[if \ |\partial_x u| > c]$$

$$= v \ll V \,[if \ |\partial_x u| < c]$$

Model A: $P(\partial_x u) \propto \exp[-c(\partial_x u)^4]$



Model A

Model B

IV. Conclusion

- Powerful theory of intermittency (esp. PDF tails)
- Agreement with simulations and experiments
- Much scope for extension

  - structures + turbulence
  - multi-structures/multi-instantons
  - forcing with finite correlation time
  - consistent incorporation of instabilities
  - hope for a diverse scaling

- Works on blob transport and consistent modelling of zonal flows in ITG (Anderson and Kim 08)